3. Economy, core and competitive equilibrium

Denote by E the commodity space. Throughout this section the commodity space E will be an ordered Banach space [see Aliprantis and Burkinshaw (1985)]. We will denote by E_+ and E_- the positive and negative cones of E, respectively.

An *economy* ε is a quadruple $[(T,\tau,\mu),X,>,e]$, where

(1) (T,τ,μ) is a *measure space of agents*,
(2) $X\colon T\to 2^E$ is a *consumption correspondence*,
(3) $>_t\subset X(t)\times X(t)$ is the *preference relation*[3] of agent t, and
(4) $e\colon T\to E$ is the *initial endowment*, where e is Bochner integrable and $e(t)\in X(t)$ for all $t\in T$.

An *allocation* for the economy ε is a Bochner integrable function $x\colon T\to E_+$. An allocation x is said to be *feasible* if $\int_T x(t)\,\mathrm{d}\mu(t)=\int_T e(t)\,\mathrm{d}\mu(t)$. A *coalition* S is an element of τ such that $\mu(S)>0$. The coalition S can *improve upon* the allocation x if there exists allocation g such that

(i) $g(t)>_t x(t)$ μ-a.e. in S, and
(ii) $\int_S g(t)\,\mathrm{d}\mu(t)=\int_S e(t)\,\mathrm{d}\mu(t)$.

The set of all feasible allocations for the economy ε that no coalition can improve upon is called the *core* of the economy ε and is denoted by $C(\varepsilon)$.

An allocation x and a price $p\in E^*_+\setminus\{0\}$ are said to be a *competitive equilibrium* (or a *Walras equilibrium*) for the economy ε, if

(i) $x(t)$ is a maximal element of $>_t$ in the budget set

$$\{y\in X(t)\colon p\cdot y\leq p\cdot e(t)\}\,\mu-\text{a.e.,}\quad \text{and}$$

(ii) $\int_T x(t)\,\mathrm{d}\mu(t)=\int_T e(t)\,\mathrm{d}\mu(t)$.

We denote by $W(\varepsilon)$ the set of all competitive equilibria for the economy ε.

4. Core–Walras equivalence in ordered Banach spaces whose positive cone has a non-empty norm interior

We begin by stating some assumptions needed for the proof of our core–Walras equivalence result.

A.1. E is an ordered separable Banach space whose positive cone E_+ has a non-empty norm interior, i.e., $\operatorname{int}E_+\neq\emptyset$.

[3] $>$ is defined to be the asymmetric part of the weak preference relation $\succsim$, i.e., we say that $x>y$ if and only if $x\succsim y$ and not $y\succsim x$. This is not needed for Theorem 4.1. However, it is used in the proof of Theorem 6.1.