$G_\phi = \{(x, y) \in X \times Y: y \in \phi(x)\}$. Let (T, τ, μ) be a finite measure space, and X be a Banach space [for a treatment of infinite dimensional vector spaces see Aliprantis and Burkinshaw (1978, 1985)]. The correspondence $\phi: T \to 2^Y$ is said to have a *measureable graph* if $G_\phi \in \tau \otimes \beta(X)$, where $\beta(X)$ denotes the Borel σ-algebra on X and $\otimes$ denotes product σ-algebra. A function $f: T \to X$ is called *simple* if there exist $x_1, x_2, \ldots, x_n$ in X and a_1, $a_2, \ldots, a_n$ in τ such that

$$f = \sum_{i=1}^{n} x_i \chi_{a_i} \quad \text{where} \quad \chi_{a_i}(t) = 1 \quad \text{if} \quad t \in a_i \quad \text{and}$$

$$\chi_{a_i}(t) = 0 \quad \text{if} \quad t \notin a_i.$$

A function $f: T \to X$ is said to be *μ-measurable* if there exists a sequence of simple functions $f_n: T \to X$ such that $\lim_{n\to\infty} \|f_n(t) - f(t)\| = 0$ μ-a.e. A *μ-measurable* function $f: T \to X$ is said to be *Bochner integrable* if there exists a sequence of simple functions $\{f_n: n = 1, 2, \ldots\}$ such that

$$\lim_{n\to\infty} \int_T \|f_n(t) - f(t)\| \mathrm{d}\mu(t) = 0.$$

In this case we define for each $E \in \tau$ the integral to be $\int_E f(t)\,\mathrm{d}\mu(t) = \lim_{n\to\infty} \int_E f_n(t)\,\mathrm{d}\mu(t)$. It can easily be shown [see Diestel and Uhl (1977, p. 45)] that if $f: T \to X$ is a μ-measurable function then f is Bochner integrable if and only if $\int_T \|f(t)\|\,\mathrm{d}\mu(t) < \infty$. We denote by $L_1(\mu, X)$ the space of equivalence classes of X-valued Bochner integrable functions $x: T \to X$ normed by $\|x\| = \int_T \|x(t)\|\,\mathrm{d}\mu(t)$. Moreover, we denote by S_ϕ the *set of all X-valued Bochner integrable selections from the correspondence* $\phi: T \to 2^X$, i.e.

$$S_\phi = \{x \in L_1(\mu, X): x(t) \in \phi(t)\, \mu\text{-a.e.}\}.$$

As in Aumann (1966), the *integral of the correspondence* $\phi: T \to 2^X$ is defined as

$$\int_T \phi(t)\,\mathrm{d}\mu(t) = \left\{ \int_T x(t)\,\mathrm{d}\mu(t): x \in S_\phi \right\}.$$

In the sequel we will denote the above integral by

$$\int \phi \quad \text{or} \quad \int_T \phi.$$