

(1986), which in turn is related to the condition of uniform properness in Mas-Colell (1986).<sup>2</sup> This assumption is essentially a bound on the marginal rates of substitution, and in practice turns out to be quite weak. For instance it is automatically satisfied whenever preferences are monotone and the positive cone of the commodity space has a non-empty (norm) interior. Hence, this assumption is implicit in any infinite dimensional commodity space whose positive cone has a non-empty interior, and is automatically satisfied in the finite dimensional work of Aumann (1964) and Hildenbrand (1974). We also wish to note that in addition to the assumption of an extremely desirable commodity, the lattice structure of the commodity space will play a crucial role in our analysis.

The remainder of the paper is organized as follows: Section 2 contains notation and definitions. The economic model is outlined in section 3. In section 4 we state and prove a core-Walras equivalence theorem for an ordered separable Banach space of commodities, whose positive cone has a non-empty (norm) interior. The failure of this result for spaces whose positive cone has an empty interior is established in section 5. In section 6, we prove a core-Walras equivalence result for a commodity space which can be any arbitrary separable Banach lattice, whose positive cone may have an empty (norm) interior. Finally, some concluding remarks are given in section 7.

## 2. Notation and definitions

### 2.1. Notation

$R^l$  denotes the  $l$ -fold Cartesian product of the set of real numbers  $R$ .

$\text{int } A$  denotes the interior of the set  $A$ .

$2^A$  denotes the set of all non-empty subsets of the set  $A$ .

$\emptyset$  denotes the empty set.

$/$  denotes the set theoretic subtraction.

$\text{dist}$  denotes distance.

If  $A \subset X$  where  $X$  is a Banach space,  $\text{cl } A$  denotes the norm closure of  $A$ . If  $X$  is a Banach space its dual is the space  $X^*$  of all continuous linear functionals on  $X$ .

If  $q \in X^*$  and  $y \in X$  the value of  $q$  at  $y$  is denoted by  $q \cdot y$ .

### 2.2. Definitions

Let  $X, Y$  be sets. The *graph* of the correspondence  $\phi: X \rightarrow 2^Y$  is denoted by

<sup>2</sup>It should be noted that a precursor of the assumption of uniform properness is in Chichilnisky and Kalman (1980). In particular, in order to apply Hahn-Banach-type separation theorems in spaces whose positive cone has an empty interior, they introduced a related assumption with that of uniform properness used by Mas-Colell.