and S_H are stability functions for momentum and scalars. q^2 and $q^2 l$ are calculated prognostically through the following equations (written in z coordinates for simplicity here),

$$\frac{\partial q^2}{\partial t} + v \cdot \nabla q^2 = 2K_v \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] + \frac{2g}{\rho_o} K_\rho \frac{\partial \sigma_\theta}{\partial z} - \frac{2q^3}{B_1 l} + \frac{\partial}{\partial z} \left[K_q \frac{\partial q^2}{\partial z} \right] \tag{3}$$

$$\frac{\partial q^2 l}{\partial t} + v \cdot \nabla q l = E_1 l \left\{ K_v \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] + \frac{g}{\rho_o} K_\rho \frac{\partial \sigma_\theta}{\partial z} \right\} - \frac{q^3}{B_1} W + \frac{\partial}{\partial z} \left[K_q \frac{\partial q^2 l}{\partial z} \right] \tag{4}$$

[12] In these equations K_q is estimated as $0.41 K_v$. W is a wall proximity function expressed as,

$$W = 1 + \frac{E_2 \tilde{l}}{\kappa^2} \left[\frac{1}{\eta - z} + \frac{1}{H + z} \right]^2 \tag{5}$$

The stability functions in the expressions for eddy viscosity and diffusivity are

$$S_H = \frac{A_2 \left(1 - 6 A_1 B_1^{-1} \right)}{1 - \left(3 A_2 B_2 (1 - C_3) + 18 A_1 A_2 \right) G_H} \tag{6}$$

$$S_M = \frac{A_1 \left(1 - 3C_1 - 6 A_1 B_1^{-1} \right) - S_H \left[G_H \left(18 A_1^2 + 9 A_1 A_2 (1 - C_2) \right) \right]}{1 - 9 A_1 A_2 G_H}. \tag{7}$$

Parameters in the above equations are set as

$$A_1 = 0.92, A_2 = 0.74, \tag{8}$$

$$B_1 = 16.6, B_2 = 10.1, \tag{9}$$

$$C_1 = 0.08, C_2 = 0.7, C_3 = 0.2, \tag{10}$$

$$E_1 = 1.8, E_2 = 1.33 \tag{11}$$

and

$$G_H = \min \left(\frac{-\tilde{l}^2 N^2}{q^2}, 0.028 \right). \tag{12}$$

[13] The turbulent length scale contained in the $q^2 l$ equation differs from $\tilde{l}$ which is used in the estimates for K_v and K_ρ. As suggested by *Galperin et al.* [1988] a maximum is placed on the length scale of eddies that contribute to vertical mixing in stably stratified conditions. This limit is

$$\tilde{l} = \min \left(l, \frac{0.53 q}{N} \right), \tag{13}$$

where N is the buoyancy frequency.

[14] The total vertical mixing coefficients are the sum of the turbulent contribution plus a constant "background" level. For these experiments the background coefficients are set as $\nu_M = 10^{-5}$ m^2 s^{-1}, $\nu_H = 10^{-6}$ m^2 s^{-1} and $\nu_q = 10^{-6}$ m^2 s^{-1}.

[15] A third-order upwind scheme is utilized to time step the advection terms [*Shchepetkin and McWilliams*, 1998]. The diffusion terms are time stepped semi-implicitly using a Crank-Nicholson formulation. The turbulent dissipation terms in both the q^2 and $q^2 l$ equations are discretized in time following the implementation in POM [*Blumberg and Mellor*, 1987]. The q^3 in this term is expressed as the product of $q|^n$ (the value calculated at the previous time step n) multiplied by $q^2|^{n+1}$ (the current value being estimated at time step $n + 1$). Because this term involves "future" values of the variable it also must be solved for implicitly.

[16] The turbulent production terms involve estimates of the square of the vertical shear in the horizontal velocity and the vertical potential density gradient. It is often necessary to apply Shapiro filters horizontally to these fields to avoid extremely noisy results. This approach is taken here for the two-dimensional simulations.

[17] As a final note on the M-Y formulation, it is worth pointing out two features of the parameterization that will play an important role in this study. (1) Here q^2 and $q^2 l$ diffuse vertically such that mixing can exist above background levels in portions of the water column where the mean shear and stratification do not support production of turbulence locally. (2) Buoyant suppression exceeds shear production of turbulence in a stably stratified fluid when the gradient Richardson number (Ri_g) exceeds approximately 0.21, where

$$Ri_g = \frac{N^2}{Sh^2} \tag{14}$$

in which Sh^2 is the square of the vertical shear in the horizontal velocity.

2.2. Large, McWilliams, and Doney Parameterization

2.2.1. Basic Formulation

[18] A description of the KPP formulation as it applies to this study will be discussed next. Several aspects of the full formulation which are not relevant to this study, will be excluded. These include the counter-gradient flux term and other portions of the boundary layer formulation associated with nonneutral surface buoyancy fluxes. The double-diffusive mixing parameterization for the interior is also excluded.

[19] The K profile parameterization of *Large et al.* [1994] matches separate parameterizations for vertical mixing of the surface boundary layer and the ocean interior. A formulation based on boundary layer similarity theory is applied in the water column above a calculated depth (h_{sbl}). This is matched at the base of the boundary layer with mixing formulations to account for local shear and internal wave effects.

[20] Viscosity and diffusivities at model levels above h_{sbl} are expressed as the product of the length scale h_{sbl}, a turbulent velocity scale w_f, and a nondimensional shape function G_f,

$$K_x = h_{sbl} w_f(\sigma) G_f(\sigma) \tag{15}$$