which need not be transitive or complete can be replaced by weak preference relations which need not be transitive or complete. Using their methods one can obtain core existence results for weak preference relations which need not be transitive or complete.

**Remark 7.5.** A rather more natural definition of the core with interdependent preferences is what Aumann (1964) calls strong equilibrium. One may define a *strong equilibrium allocation* of $\mathcal{E} = \{(X_i, P_i, e_i) : i \in I\}$ as a vector $x = (x_1, \ldots, x_N) \in X$ such that

(i) $x$ is feasible, and
(ii) it is not true that there exist $S \subset I$ and $(y_i)_{i \in S} \prod_{i \in S} X_i$ such that for all $i \in S$, $\sum_{i \in S} y_i = \sum_{i \in S} e_i$, $\sum_{i \notin S} x_i = \sum_{i \notin S} e_i$, and $(y^S, x^{I \setminus S}) \in P_i(x_1, \ldots, x_N)$.

However, Scarf (1967, p. 180) showed that even with stronger conditions than those used in Theorem 4.2, the $\beta$-core (recall that the set of strong equilibrium allocations is a subset of the $\beta$-core) may be empty and therefore the set of strong equilibrium allocations may be empty as well.

# Appendix

Fan's (1962) extension of the K-K-M Lemma to Hausdorff linear topological spaces was based on the finite dimensional K-K-M result. This way of proving an infinite dimensional result by considering its trace on finite dimensions, sometimes simplifies the arguments considerably. Indeed this method of proof was adopted by Fan (1952) to extend the Kakutani fixed point to Hausdorff locally convex linear topological spaces. We now provide an alternative proof of the K-K-M-F theorem which is similar in spirit with that of Fan but makes use of the Brouwer fixed point theorem. In addition to the fact that our proof is very intuitive it turns out to be elementary. Notice that in finite dimensions the Brouwer fixed point can be used to derive the K-K-M theorem,[3] the Sperner Lemma and the Kakutani fixed point theorem. In that sense Brouwer's result may be considered as a milestone in Fixed Point Theory.

[3] A proof that the Brouwer fixed point theorem implies the K-K-M Lemma is $\mathbb{R}^\ell$ is given by Ichiichi (1981a) who attributes the argument to K. C. Border and E. Green. Although our proof is more involved than that in Ichiichi (1981) the idea is essentially the same.