are $X_1 = X_2 = C[0,1]^+$, *i.e., the positive cone of* $C[0,1]$. *Their utility functions and their initial endowments are given as follows:*

$$u_1(x) = \int_0^1 tx(t)\,dt,$$

$$u_2(x) = \int_0^1 (1-t)x(t)\,dt,$$

and

$$e_1 = e_2 = \frac{1}{2}.$$

Notice that utility functions are norm continuous, concave, and monotone. However, there is no individually rational Pareto optimal allocation, i.e., the extreme core which coincides with the selfish core is empty. (Of course there are two Pareto optimal allocations which are not individually rational, i.e., give all the initial endowment to either agent 1 or agent 2.) The non-existence of extreme core allocations lies on the fact that the set of all feasible allocations (which is norm closed and bounded) is not norm compact (notice that consumption sets are not norm compact). Thus, the proofs of Theorems 4.1 and 4.2 or Corollary 4.1 do not go through. The same difficulty occurs in the Araujo-Mas-Colell example [see for instance Araujo (1974, Theorem 3)]. In particular there are two agents whose preferences are norm continuous monotone convex but consumption sets are not norm compact. Hence, the above examples have violated assumption (4.7) of Corollary 4.1 and assumptions (4.3) and (4.6) of Theorems 4.1 and 4.2 respectively. Consequently, the conclusion to be drawn is that if the set of all feasible allocations is compact in a topology which is at least as strong as the topology in which preferences are continuous, then extreme α-core allocations always exist.

The intuition behind the above conclusion is quite simple. In particular, the maximal elements result (Theorem 3.3) is used to prove the existence of extreme α-core allocations (Theorem 4.1). However, if in Theorem 3.3 the preference correspondence $P : X \to 2^X$ has open lower sections in a topology which is stronger than the topology in which the set X is compact, then Theorem 3.3 fails and a fortiori Corollary 3.1 fails as well. The following example illustrates this.