Denote the convergent subnet by $\{(x_1^{f(m)}, \ldots, x_N^{f(m)}) : m \in M\}$ where M is a set directed by "$\geq$." First of all we know that $\sum_{i\in I} x_i^{f(m)} = \sum_{i\in I} e_i$ for all $m \in M$. Since the vectors $x_i^{f(m)}$ converge to x_i^* in the compatible topology τ, and τ is a vector space topology we conclude that $\sum_{i\in I} x_i^* = \sum_{i\in I} e_i$, i.e., $x^* = (x_1^*, \ldots, x_N^*)$ is a feasible allocation for the economy $\mathcal{E}$. To complete the proof we must show that:

(5.9) it is not true that there exist $S \subset I$ and $(y_i)_{i\in S} \in \prod_{i\in S} X_i$ such that $\sum_{i\in S} y_i = \sum_{i\in S} e_i$ and $y_i \in \bar{P}_i(x_i^*)$ for all $i \in S$.

Suppose otherwise, i.e., there exist $S \subset I$ and $(y_i)_{i\in S} \in \prod_{i\in S} X_i$ such that $\sum_{i\in S} y_i = \sum_{i\in S} e_i$ and $y_i \in \bar{P}_i(x_i^*)$ for all $i \in S$. Since $x_i^{f(m)}$ converges to x_i^* in the compatible topology τ and by assumption (4.6) $\bar{P}_i$ has a τ-open graph, there exists $m_1 \in M$ such that $y_i \in \bar{P}_i(x_i^{f(m)})$ for all $m \geq m_1$ and all $i \in S$. Choose $m_2 \geq m_1$ so that, if $m \geq m_2$, $y_i \in X_i^{f(m)}$ for all $i \in S$. Then $y_i \in \bar{P}_i^{f(m)}(x_i^{f(m)})$ for all $m \geq m_2$, all $i \in S$, and clearly $\sum_{i\in S} y_i = \sum_{i\in S} e_i$. But this contradicts the fact that $x^{f(m)}$ is a core allocation of the economy $\mathcal{E}^{f(m)}$. Hence (5.9) is satisfied and this completes the proof of the theorem.

5.3 Proof of Corollary 4.1. It follows from assumption (4.7) that the set of all feasible allocations F is compact. Therefore, an identical argument with that used in Theorem 4.1 can be adopted to complete the proof of the Corollary.

6. Examples

We can now turn to some known pathological examples in the literature and see what goes wrong in infinite dimensions. In particular, Araujo (1985), Mas-Colell (1986) and Jones (1986) illustrated by means of simple examples the difficulties in obtaining existence of extreme core allocations in an infinite dimensional commodity setting. The following simple example due to Jones (1986) may be used to illustrate these difficulties.

Example 6.1. *Consider an economy with two agents, i.e., $I = \{1, 2\}$. The commodity space is $L = C[0,1]$, i.e., the space of continuous functions on the interval $[0,1]$ under the supnorm. The consumption sets*