sections in F, then $\theta(z) = \bigcap_{i \in I} \Phi_i(z) \subset \Phi_j(z)$ for all $z \in A$. If (b) holds then $\theta(x) = \bigcap_{i \in S_x} \Phi_i(x) \neq \emptyset$. Choose $e \in \theta(x)$. Then $e \in \Phi_i(x)$ for all $i \in S_x$ which implies that $e \in P_i(x)$ for all $i \in S_x$. Fix an agent j in S_x. Since P_j has τ-open lower sections in F there exists a neighborhood of x, N_x such that $e \in P_j(z)$ for all $z \in N_x$. But then $j \in S_z$ for all $z \in N_x$. Consequently, $\theta(z) = \bigcap_{i \in S_z} \Phi_i(z) \subset \Phi_j(z)$ for all $z \in N_x$. Therefore, θ is $\mathcal{L}$-majorized. By Corollary 3.1 there exists $x^* \in F$ such that $\theta(x^*) = \emptyset$, a contradiction to (5.6). Since we have obtained a contradiction to our supposition that $\mathcal{C}_e(\mathcal{E}) = \emptyset$ the proof of the Theorem is complete.

5.2 Proof of Theorem 4.2. Let $\mathcal{F}$ be the set of all finite dimensional subspaces of L containing the initial endowments. For each $f \in \mathcal{F}$ and for each $i \in I$ define the consumption set X_i^f and the preference correspondence $\bar{P}_i^f : X_i^f \to 2^{X_i^f}$ by

$$X_i^f = X_i \cap f$$

$$\bar{P}_i^f(x_i) = \bar{P}_i(x_i) \cap f.$$

We now have an economy $\mathcal{E}^f = \{(X_i^f, \bar{P}_i^f, e_i) : i = 1, \ldots, N\}$, in a finite dimensional commodity space. It can be checked that each economy $\mathcal{E}^f$ satisfies all the conditions of Border's Proposition (1984, p. 1540), and consequently for each $f \in \mathcal{F}$, $\mathcal{C}_s(\mathcal{E}^f) \neq \emptyset$, i.e., there exists $x^f = (x_1^f, \ldots, x_N^f)$ in $\prod_{i \in I} X_i^f$ such that:

(5.7) $\sum_{i \in I} x_i^f = \sum_{i \in I} e_i$, and

(5.8) it is not true that there exist $S \subset I$ and $(y_i)_{i \in S} \in \prod_{i \in S} X_i^f$ such that $\sum_{i \in S} y_i = \sum_{i \in S} e_i$ and $y_i \in \bar{P}_i^f(x_i^f)$ for all $i \in S$.

From (5.7) it follows that for each $f \in \mathcal{F}$

$$0 \leq \sum_{i \in I} x_i^f = \sum_{i \in I} e_i = e \leq Ne.$$

Hence for each $f \in \mathcal{F}$ the vectors x_i^f lie on the order interval $[0, Ne]$, which is τ-compact. Direct the set $\mathcal{F}$ by inclusion so that $\{(x_1^f, \ldots, x_N^f) : f \in \mathcal{F}\}$ forms a net in $L \times L \times \cdots \times L$. Since all the vectors x_i^f belong to the order interval $[0, Ne]$ which is τ-compact, the net $\{(x_1^f, \ldots, x_N^f) : f \in \mathcal{F}\}$ has a subnet which converges in the compatible topology τ, to some vector $x_1^*, \ldots, x_N^*$ in $[0, Ne]$. We must show that $x_1^*, \ldots, x_N^*$ is a core allocation for the economy $\mathcal{E}$.