compact [see Aliprantis and Burkinshaw (1985, Theorem 9.20)]. Finally, if the commodity space is the space of real sequences ℓ_p, $1 \leq p < \infty$ the compatible topology will be the norm topology. This follows from the standard result that order intervals in ℓ_p, $1 \leq p < \infty$ are norm compact [see for instance Yannelis and Zame (1984, Theorem 10.1, p. 48)].

It may be instructive to compare our continuity assumption (4.3) with that of Araujo (1985) [or Berninghaus (1977)] whose commodity space is ℓ_∞ (or L_∞), with consumption sets $X = \ell_\infty^+$ (or L_∞^+).

In Araujo (1985), preferences are given by a weak preference relation $\succsim$ which is reflexive, transitive, complete. Assume that $\succsim$ satisfies:

(i) the set $\{y \in X : y \succsim x\}$ is Mackey closed in X and convex for each $x \in X$,

(ii) the set $\{x \in X : y \succsim x\}$ is norm closed in X for each $y \in X$.

If we let P be the strict preference relation induced by $\succsim$, then $P(x) = X \setminus \{y \in X : x \succsim y\}$ and $P^{-1}(y) = \{x \in X : y \in P(x)\} = X \setminus \{x \in X : x \succsim y\}$. Therefore, for each $x \in X$, $P(x)$ is norm open in X and for each $y \in X$, $P^{-1}(y)$ is Mackey open in X. However, since by the Mackey-Arens Theorem [see for instance Bewley (1972, p. 352, (8))] the Mackey topology coincides with the weak* topology on closed convex sets, it follows that the set $\{y \in X : y \succsim x\}$ is weak* closed in X and consequently $P^{-1}(y)$ is weak* open in X. Therefore, since in L_∞ (or ℓ_∞) the compatible topology is the weak* topology, the continuity assumption (4.3) in Theorem 4.1, for $L = L_\infty$ is not stronger than the ones of Araujo's (1985) (or Berninghaus' (1977)) continuity assumptions, who require that the set $\{y \in X : y \succsim x\}$ is Mackey (weak*) closed in X for every $x \in X$. Hence, Theorem 4.1 can be considered as a generalization of the existence results of Araujo (1985) and Berninghaus (1977). Specifically, the commodity space can be any arbitrary ordered linear topological space and preferences need not be transitive, complete or convex and may be interdependent.

5. Proof of the Theorems

5.1 Proof of Theorem 4.1. Suppose otherwise, i.e., $\mathcal{C}_e(\mathcal{E}) = \emptyset$, then for all $x \in F$ either