in L, where L is endowed with the compatible topology τ, satisfying for each $i \in I$ the following assumptions:

(4.1) $X_i = L^+$, *(L^+ denotes the positive cone of L),*

(4.2) $x \notin \text{con}\, P_i(x)$ *for all* $x \in X$,

(4.3) P_i *has τ-open lower sections, i.e., for each $y \in X$ the set $\{x \in X : y \in P_i(x)\}$ is τ-open in X.*

Then there exists an extreme α-core allocation of $\mathcal{E}$, i.e., $\mathcal{C}_e(\mathcal{E}) \neq \emptyset$.

Theorem 4.2. *Let $\mathcal{E} = \{(X_i, \bar{P}_i, e_i) : i \in I\}$ be an exchange economy in L, where L is endowed with the compatible topology τ, satisfying for each $i \in I$ the following assumptions:*

(4.4) $X_i = L^+$,

(4.5) $x_i \notin \text{con}\, \bar{P}_i(x_i)$ *for all* $x_i \in X_i$,

(4.6) $\bar{P}_i$ *has a τ-open graph, i.e., the set $\{(x_i, y_i) \in X_i \times X_i : y_i \in \bar{P}_i(x_i)\}$ is τ-open in $X_i \times X_i$.*

Then there exists a selfish core allocation of $\mathcal{E}$, i.e., $\mathcal{C}_s(\mathcal{E}) \neq \emptyset$.

Corollary 4.1. *Let $\mathcal{E} = \{(X_i, P_i, e_i) : i \in I\}$ be an exchange economy in L, satisfying the following assumptions:*

(4.7) X_i *is a nonempty, convex, compact subset of L,*

(4.8) $x \notin \text{con}\, P(x)$ *for all* $x \in X$,

(4.9) P_i *has open lower sections.*

Then there exists an extreme α-core allocation of $\mathcal{E}$, i.e., $\mathcal{C}_e(\mathcal{E}) \neq \emptyset$.

Corollary 4.2. *Since for $I = \{1,2\}$, $\mathcal{C}(\mathcal{E}) = \mathcal{C}_e(\mathcal{E})$ it follows from Theorem 4.1 that $\mathcal{C}(\mathcal{E}) \neq \emptyset$.*

Remark 4.1. Notice that if in Theorem 4.1 and Corollary 4.1 we had selfish preferences, i.e., $\bar{P}_i : X_i \to 2^{X_i}$ the arguments in the proofs (see Section 5) remain unaffected. In fact, define $P_i : X \to 2^X$ by $P_i(x) = \bar{P}_i(x_i) \times \prod_{j \neq i} X_j$, then the proofs go through with no modification.

4.6 Discussion of the Assumptions.

Let us now discuss the assumptions in Theorems 4.1 and 4.2.

First notice that (4.1) is identical with (4.4), and (4.2) is essentially the same with (4.5). Assumption (4.1) is quite standard in equilibrium theory and needs no explanation. Assumption (4.2) is a very weak form