An *extreme α-core allocation* of $\mathcal{E}$ is a vector $x = (x_1, \ldots, x_N) \in \prod_{i \in I} X_i$ which satisfies individual rationality and Pareto optimality.

Denote by $\mathcal{C}_e(\mathcal{E})$ the *set of all extreme α-core allocations for the economy* $\mathcal{E}$. Notice that the concept of extreme α-core allocations takes into account only two extreme coalitions, i.e., the grant coalition and the coalitions of one agent alone. Therefore, it is clear that the set of all extreme α-core allocations for $\mathcal{E}$ is bigger than the set of all α-core allocations for $\mathcal{E}$, i.e., $\mathcal{C}(\mathcal{E}) \subseteq \mathcal{C}_e(\mathcal{E})$. However, *it is easy to see that in a two person economy, i.e., when* $|I| = 2$, $\mathcal{C}(\mathcal{E}) = \mathcal{C}_e(\mathcal{E})$.

Finally, if preferences are "selfish," i.e., $P_i : X_i \to 2^{X_i}$ is defined by $P_i(x_i) = \{y_i \in X_i : y_i \mathcal{P}_i x_i\}$, we will call an individually rational Pareto optimal allocation, an *extreme core allocation.*

4.4 The Selfish Core. Let $\mathcal{E} = \{(X_i, \bar{P}_i, e_i) : i = 1, \ldots, N\}$ be an exchange economy, where $\bar{P}_i : X_i \to 2^{X_i}$ is defined by $\bar{P}_i(x_i) = \{y_i \in X_i : y_i \mathcal{P}_i x_i\}$. Notice that preferences are not interdependent. In this framework we may define the notion of selfish core or simply core as follows:

A *selfish core* (or *core*) *allocation* of $\mathcal{E}$ is a vector $x = (x_1, \ldots, x_N) \in X$ such that:

(i) $x \in F$, and
(ii) it is not true that there exist $S \subset I$ and $(y_i)_{i \in S} \in \prod_{i \in S} X_i$ such that $\sum_{i \in S} y_i = \sum_{i \in S} e_i$ and $y_i \in \bar{P}_i(x_i)$ for all $i \in S$.

The above notion of core is the one extensively used in equilibrium analysis. In fact, this is the notion of core used recently in Border (1984) as well. Denote by $\mathcal{C}_s(\mathcal{E})$ the *set of all selfish core allocations for* $\mathcal{E}$.

4.5 Theorems. Before we state our two main results we will need the following definition.

Definition 4.1. *A Hausdorff topology* τ, *on an ordered Hausdorff linear topological space* L, *will be called compatible if:*

(a) τ *is weaker than the Hausdorff topology of* L;
(b) τ *is a vector space topology (i.e., the vector space operations on* L *are continuous in the topology* τ*);*
(c) all order intervals $[0, y] = \{z \in L : 0 \leq z \leq y\}$ *in* L *are* τ*-compact.*

Theorem 4.1. *Let* $\mathcal{E} = \{(X_i, P_i, e_i) : i \in I\}$ *be an exchange economy*