of all feasible allocations, i.e., $F = \{x \in X : \sum_{i \in I} x_i = \sum_{i \in I} e_i\}$. Notice that we have allowed for interdependent preferences. In this framework $y \in P_i(x)$ means that agent i strictly prefers the allocation y to x. More simply one may define $P_i : X \to 2^X$ by $P_i(x_1, \ldots, x_N) = \{y \in X : (y_1, \ldots, y_N)\mathcal{P}_i(x_1, \ldots, x_N)\}$.

4.2 The α-Core. If $S \subset I$ then $(y^S, x^{I \setminus S})$ denotes the vector z in X such that:

$$z_i = \begin{cases} y_i & \text{if } i \in S \\ x_i & \text{if } i \notin S. \end{cases}$$

An *α-core allocation* of $\mathcal{E}$ is a vector $x = (x_1, \ldots, x_N) \in X$ such that:

(i) $x \in F$, and

(ii) it is not true that there exist $S \subset I$ and $(y_i)_{i \in S} \in \prod_{i \in S} X_i$ such that $\sum_{i \in S} y_i = \sum_{i \in S} e_i$, and $(y^S, z^{I \setminus S}) \in P_i(x_1, \ldots, x_N)$ for all $i \in S$ and for any $z \in \prod_{i \notin S} X_i$, $\sum_{i \notin S} z_i = \sum_{i \notin S} e_i$.

In other words an α-core allocation for the economy $\mathcal{E}$ must satisfy two conditions. First it must be feasible and secondly, no coalition of agents can redistribute their initial endowments and make all its members better off, once the complementary coalition chooses to redistribute its initial endowment. For a game in normal form, the notion of α-core was introduced in Aumann (1964). It was also used by Scarf (1971) who proved the nonemptiness of the α-core for an n-person game with a finite dimensional strategy space, where each agent's preferences were assumed to be transitive, complete, and continuous.

The *set of all α-core allocations for the economy* $\mathcal{E}$ is denoted by $\mathcal{C}(\mathcal{E})$.

4.3 The Extreme α-Core. If $i \in I$, then (y_i, z^{I^i}) denotes the vector w in X such that:

$$w_j = \begin{cases} y_j & \text{if } j = i \\ z_j & \text{if } j \neq i. \end{cases}$$

An allocation $x \in X$ is said to be *individually rational* if:

(i) $x \in F$, and

(ii) for all $i \in I$, it is not true that $e \in P_i(x)$.

An allocation $x \in X$ is said to be *Pareto optimal* if:

(i) $x \in F$, and

(ii) there is no $y \in F$ such that $y \in P_i(x)$ for all $i \in I$.