

Definition 3.1. Let X be a subset of a linear topological space. A correspondence $\phi : X \rightarrow 2^X$ is said to be of class \mathcal{L} , if

- (i) $x \notin \text{con } \phi(x)$ for all $x \in X$,
- (ii) ϕ has open lower sections.

Let $\psi : X \rightarrow 2^X$ be a correspondence. The correspondence $\phi_x : X \rightarrow 2^X$ is an \mathcal{L} -majorant of ψ at x if ϕ_x is of class \mathcal{L} and there is an open neighborhood of x denoted by N_x such that for all $z \in N_x$, $\psi(z) \subset \phi_x(z)$. The correspondence $\psi : X \rightarrow 2^X$ is \mathcal{L} -majorized if for each $x \in X$ such that $\psi(x) \neq \emptyset$, there is an \mathcal{L} -majorant of ψ at x .

Corollary 3.1. Let X be a nonempty, compact, convex subset of a Hausdorff linear topological space and $\phi : X \rightarrow 2^X$ be an \mathcal{L} -majorized correspondence. Then there exists $x^* \in X$ such that $\phi(x^*) = \emptyset$.

By means of the above Corollary we will prove Theorem 4.1. We would like to emphasize the fact that Corollary 3.1 is a consequence of the Theorems of K-K-M-F and Browder. Moreover, it was pointed out in Borglin-Keiding (1976) that Corollary 3.1 yields an extension of the Kakutani fixed point to Hausdorff locally convex linear topological spaces. With those preliminary mathematical results out of the way we now turn to our core existence theorems.

4. The Main Results

4.1 The Economy. We formalize the notion of an exchange economy in the usual way. Let $I = \{1, \dots, N\}$ be a finite set of agents. For each $i \in I$, let X_i be a nonempty subset of an ordered Hausdorff linear topological space L . By an *exchange economy with N agents and a commodity space L* (or simply *an economy in L*) we mean the set $\mathcal{E} = \{(X_i, P_i, e_i) : i = 1, \dots, N\}$ of triples where,

- (a) X_i is the *consumption set* of agent i ;
- (b) $P_i : X \rightarrow 2^X$ (where $X = \prod_{i \in I} X_i$) is the *preference correspondence* of agent i ;
- (c) e_i is the *initial endowment* of agent i , where $e_i \in X_i$ for all $i \in I$.

An *allocation* is a vector $x = (x_1, \dots, x_N) \in X = \prod_{i \in I} X_i$. An allocation x is said to be *feasible* if $\sum_{i \in I} x_i = \sum_{i \in I} e_i$. Denote by F the set