we have that $\text{con}\{y_1, \ldots, y_n\} \subset \bigcup_{i=1}^{n} F(y_i)$. For otherwise, there exists $x \in \text{con}\{y_1, \ldots, y_n\}$ and $x \notin \bigcup_{i=1}^{n} F(y_i)$ which implies that $x \in \phi^{-1}(y_i)$ for all i or $y_i \in \phi(x)$ for all i and therefore $x \in \text{con}\{y_1, \ldots, y_n\} \subset \text{con}\,\phi(x) = \phi(x)$, a contradiction to $x \notin \phi(x)$ for all $x \in X$. Hence, by Theorem 3.1 $\bigcap_{y \in X} F(y) \neq \emptyset$. Let $z \in \bigcap_{y \in X} F(y)$. Then for all $y \in X$, $z \notin \phi^{-1}(y)$ which implies that $\phi(z) = \emptyset$, for some $z \in X$. But this contradicts assumption (1). Therefore there exist $x^* \in X$ such that $x^* \in \phi(x^*)$, and the proof of the Browder theorem is now complete.

3.4 Existence of Maximal Elements.

It is easy to check that Browder's fixed point theorem is equivalent to the following existence of maximal elements result.

Theorem 3.3. *Let X be a nonempty, compact, convex subset of a Hausdorff linear topological space Y and $P : X \to 2^X$ be a preference correspondence such that:*

(i) $x \notin P(x)$ for all $x \in X$
(ii) $P(x)$ is convex for all $x \in X$
(iii) P has open lower sections.

Then there exists $x^* \in X$ such that $P(x^*) = \emptyset$.

Hence, we can reach the following conclusion:

K-K-M-F $\Leftrightarrow$ *Browder Theorem*
$\Leftrightarrow$ *Existence of Maximal Elements Theorem.*

A direct consequence of the K-K-M-F or Browder theorems is the following result, whose proof can be found in Yannelis and Prabhakar (1983 p. 239, Theorem 5.1).

Theorem 3.4. *Let X be a nonempty, compact, convex subset of a Hausdorff linear topological space and $\phi : X \to 2^X$ be a correspondence having open lower section satisfying the condition that $x \notin \text{con}\,\phi(x)$ for all $x \in X$. Then there exists $x^* \in X$ such that $\phi(x^*) = \emptyset$.*

By means of Theorem 3.4 one can obtain the following Corollary [see Yannelis and Prabhakar (1983, p. 240, Corollary 5.1)] which is a generalized version of a result of Borglin-Keiding (1976, Corollary 1, p. 314). We first need to introduce a definition.