3.2 Proof of the K-K-M-F Theorem via Browder's Fixed Point Theorem. Since $F(x)$ is closed in Y for each $x \in X$ and compact for at least one $x \in X$, it suffices to prove that $\bigcap_{i=1}^{n} F(x_i) \neq \emptyset$ for every subset $\{x_1, \ldots, x_n\}$ of X. Suppose otherwise, i.e., $\bigcap_{i=1}^{n} F(x_i) = \emptyset$ for some finite subset $\{x_1, \ldots, x_n\}$ of X. Let Δ be the finite dimensional simplex spanned by the finite set $\{x_1, \ldots, x_n\}$. Since the topology induced on any finite dimensional subspace of Y by the topology of Y coincides with the Euclidean topology, Δ is homeomorphic to a Euclidean ball (Kelley and Namioka, 1963, Theorem 7.3, p. 59). Define the correspondence $\psi : \Delta \to 2^{\Delta}$ by $\psi(x) = \{y \in \Delta : x \notin F(y)\}$. Then for each $x \in \Delta$, $\psi(x)$ is nonempty. Indeed, at least one x_i, $(1 \leq i \leq n)$ is in $\psi(x)$, for otherwise $x \in \bigcap_{i=1}^{n} F(x_i)$, and so $\bigcap_{i=1}^{n} F(x_i) \neq \emptyset$. Notice that for each $y \in \Delta$, $\psi^{-1}(y) = \{x \in \Delta : y \in \psi(x)\} = \Delta \setminus \{x \in \Delta : x \in F(y)\}$. Observe that $\{x \in \Delta : x \in F(y)\} = \Delta \cap F(y)$, and this is a closed set in Δ. Hence, for each $y \in \Delta$ the set $\psi^{-1}(y)$ is open in Δ. Define the correspondence $\phi : \Delta \to 2^{\Delta}$ by $\phi(x) = \operatorname{con} \psi(x)$ for all $x \in \Delta$. Then, $\phi(x)$ is convex and nonempty for all $x \in \Delta$. Furthermore, by Lemma 5.1 in Yannelis-Prabhakar (1983) the set $\phi^{-1}(y) = \{x \in \Delta : y \in \phi(x)\}$ is open in Δ for each $y \in \Delta$. Consequently, the correspondence $\phi : \Delta \to 2^{\Delta}$ satisfies all the assumptions of Theorem 3.2. Hence, there exists $x^* \in \Delta$ such that $x^* \in \phi(x^*) = \operatorname{con} \psi(x^*)$. But, $x^* \in \operatorname{con} \psi(x^*)$ implies that there exist points $y_1, \ldots, y_m$ in Δ and real numbers $a_1, \ldots, a_m$, $a_j \geq 0$, $(1 \leq j \leq m)$, $\sum_{j=1}^{m} a_j = 1$, such that $x^* = \sum_{j=1}^{m} a_j y_j$ and $y_j \in \psi(x^*)$ for all j, a contradiction to assumption (i). Indeed, by assumption (i), for any arbitrary collection of points $\{y_1, \ldots, y_m\}$ out of X, we have that $\operatorname{con}\{y_1, \ldots, y_m\} \subset \bigcup_{i=1}^{m} F(y_i)$. Thus, if $x^* \in \operatorname{con}\{y_1, \ldots, y_m\}$, then $x^* \in \bigcup_{i=1}^{m} F(y_i)$ which implies that $x^* \in F(y_i)$ for at least one i. The above contradiction establishes that, $\bigcap_{i=1}^{n} F(x_i) \neq \emptyset$, and this completes the proof of the K-K-M-F theorem.

3.3 Proof of Browder's Fixed Point Theorem via the K-K-M-F Theorem. Suppose otherwise, i.e., for all $x \in X$, $x \notin \phi(x)$. Let for each $y \in X$, $F(y) = X \setminus \phi^{-1}(y)$. Since by assumption (3) for each $y \in X$, $\phi^{-1}(y)$ is open in X, it follows that for each $y \in X$, $F(y)$ is closed in X and obviously closed in Y since X is a compact subset of Y. Moreover, $F(y)$ is compact for each $y \in X$. It is easy to see that for any arbitrary set of points $\{y_1, \ldots, y_n\} \subset X$,