

linear topological spaces. Another simple but powerful fixed point theorem was proved by Browder (1968). Both results, in addition to their applications in mathematics, have recently proved extremely useful in economics. In fact, they have become the main technical tools to prove the existence of maximal elements and equilibria in linear topological spaces of arbitrary dimension. As a consequence, generalizations of the results of Debreu (1952), Sonnenschein (1971), Mas-Colell (1974), Gale-Mas-Colell (1975), and Shafer-Sonnenschein (1975) have been obtained [see for instance, Borglin-Keiding (1976), Yannelis-Prabhakar (1983), and Toussaint (1984)]. Since these two theorems will be the main mathematical tools used in the sequel, it is of interest to know the relationship between them. The purpose of this section is to show that Fan's generalization of the K-K-M theorem (called here K-K-M-F) theorem) can be easily derived from Browder's fixed point theorem and that the Browder fixed point theorem can be easily derived from the K-K-M-F theorem. Therefore one may consider these two results as equivalent.

The K-K-M-F theorem proved in Fan (1962) is stated below:

Theorem 3.1 (K-K-M-F). *Let X be an arbitrary convex set in a Hausdorff linear topological space Y . For each $x \in X$, let $F(x)$ be a closed set in Y such that the following two conditions are satisfied:*

- (i) *the convex hull of any finite subset $\{x_1, \dots, x_n\}$ of X is contained in $\bigcup_{i=1}^n F(x_i)$, and*
- (ii) *$F(x)$ is compact for at least one $x \in X$.*

Then $\bigcap_{x \in X} F(x) \neq \emptyset$.

We now state Browder's (1968) fixed point theorem.

Theorem 3.2 (Browder). *Let X be a compact, convex, nonempty subset of a Hausdorff linear topological space Y and $\phi : X \rightarrow 2^X$ be a correspondence such that:*

- (1) *$\phi(x)$ is nonempty for all $x \in X$,*
- (2) *$\phi(x)$ is convex for all $x \in X$,*
- (3) *for each $y \in X$, the set $\phi^{-1}(y) = \{x \in X : y \in \phi(x)\}$ is open in X , i.e., ϕ has open lower sections.*

Then there exists $x^ \in X$ such that $x^* \in \phi(x^*)$.*