

we discuss some pathological examples known in the literature. Finally, some concluding remarks are given in Section 7.

2. Notation and Definitions

2.1 Notation.

2^A denotes the set of all subsets of A .

$\text{con } A$ denotes the convex hull of the set A .

\mathbb{R}^ℓ denotes the ℓ -fold product of the set of real numbers \mathbb{R} .

$|S|$ denotes the number of elements in the set S .

If $\phi : X \rightarrow 2^Y$ is a correspondence, $\phi|_A$ denotes the restriction of ϕ to A ,
i.e., $\phi|_A : A \rightarrow 2^Y$.

\emptyset denotes the empty set.

\setminus denotes the set theoretic subtraction.

$\text{int } A$ denotes the interior of A .

If X is a linear topological space, its dual is the space X^* of all continuous linear functionals on X .

2.2 Definitions. Let X and Y be two topological spaces. Let $\phi : X \rightarrow 2^Y$ be a set-valued function (or correspondence). The set-valued function $\phi^{-1} : Y \rightarrow 2^X$ defined by $\phi^{-1}(y) = \{x \in X : y \in \phi(x)\}$ is called the *lower section* of ϕ . We say that $\phi : X \rightarrow 2^Y$ has *open lower sections* if for each $y \in Y$ the set $\phi^{-1}(y) = \{x \in X : y \in \phi(x)\}$ is open in X . A binary relation \mathcal{P} on X is a subset of $X \times X$. We read $x\mathcal{P}y$ as “ x is strictly preferred to y .” Define the correspondence $P : X \rightarrow 2^X$ by $P(x) = \{y \in X : y\mathcal{P}x\}$. We call P a *preference correspondence*, and $P(x)$ denotes its *upper section* and $P^{-1}(y)$ its *lower section*. The set-valued function $P : X \rightarrow 2^X$ has an *open graph* if the set $\{(x, y) \in X \times X : y \in P(x)\}$ is open in $X \times X$. Moreover, $P : X \rightarrow 2^X$ is said to be *lower semicontinuous* if the set $\{x \in X : P(x) \cap V \neq \emptyset\}$ is open in X for every open subset V of X . If there exists $x^* \in X$ such that $P(x^*) = \emptyset$ we say that x^* is a *maximal element* in X .

3. The K-K-M-F Lemma and the Browder Fixed Point Theorem

3.1 Theorems. Fan (1962) extended the powerful Knaster-Kuratowski-Mazurkiewicz (K-K-M) theorem from a Euclidean space to Haus-