

paper. Although the arguments of Scarf (1967, 1971) and Shapley (1973) are based on finite dimensional results, Border's (1984) proof is based on an infinite dimensional fixed point result of Fan (1969). At first glance, it seems that Border's arguments might be extended to cover infinite dimensional commodity spaces; unfortunately, a careful examination of his proof indicates that this is not possible. The problem arises from the fact that the convex hull of an upper-semicontinuous (u.s.c.) correspondence need not be u.s.c. when the dimensionality of the commodity space is infinite [see Schaefer (1971, exercise 27, p. 72)]. Consequently, in order to prove the nonemptiness of the core for an economy with infinitely many commodities and without ordered preferences different arguments than the ones used by Scarf, Shapley and Border seem to be needed. In particular, following Bewley's (1972) ideas we will prove an infinite dimensional core existence result by considering its trace in finite dimensions.

However, a different approach is adopted to prove that with interdependent preferences individually rational Pareto optimal allocations exist. In particular, the main mathematical tool that we use is an existence of maximal elements result which is a corollary of either the Knaster-Kuratowski-Mazurkiewicz (K-K-M) Lemma as extended by Fan (1962) or the Browder (1968) fixed point theorem. In fact, we will show that these two remarkable technical theorems turn out to be equivalent in the sense that each one can be derived from the other. It should be noted that the idea of using maximal elements results to prove optima goes back to Debreu (1959, p. 92). The same idea was also used in Hildenbrand (1974, Theorem 3, p. 230) and Berninghaus (1977, Theorem 1, p. 283). However, the assumption that preferences are transitive and complete and consequently representable by utility functions is crucial to their arguments. It turns out, that allowing simultaneously, preferences to be nonordered and the dimensionality of the commodity space to be infinite, rather powerful fixed point results seem to be needed. It is exactly for this reason that we make use of the theorems of Fan (1962) and Browder (1968).

The paper is organized in the following way. Section 2 contains some notation and definitions. Section 3 shows the equivalence between the K-K-M Lemma as extended by Fan and the Browder fixed point theorem. The main results of the paper, i.e., core existence theorems, are stated in Section 4 and their proofs are given in Section 5. In Section 6