

at the level of generality established for competitive equilibrium existence theorems.

The first core existence result for an economy was proved in Scarf (1967, 1971). He required agents' preferences to be transitive and complete. Border (1984) recently generalized this result to allow for preferences which need not be transitive or complete. Both authors obtain their results for economies with a finite dimensional commodity space, and follow a common argument: First, they establish that the core of a balanced non-side payment game is nonempty; and second, they show the nonemptiness of the core of an economy by showing that the game derived from an economy is balanced.<sup>1</sup>

Recently, several nonexistence core results have been given for infinite dimensional commodity spaces [see for instance Araujo (1985) and Mas-Colell (1986)]. In particular, these authors have shown by means of counter-examples that in an infinite dimensional commodity space, where agents' preferences are representable by very well-behaved utility functions, one can not necessarily even expect individually rational Pareto optimal allocations to exist. Therefore, the question is raised under what conditions can core existence results be obtained in an infinite dimensional commodity space. The purpose of this paper is to show that in any ordered Hausdorff linear topological space, core allocations exist under very mild assumptions. In particular, agents' preferences need not be ordered, monotone or nonsaturated. Indeed, under these assumptions even a quasi-equilibrium need not exist. Moreover, we show that in any ordered Hausdorff linear topological space, individually rational Pareto optimal allocations exist, even if preferences are interdependent and may not be ordered, monotone or nonsaturated.

It may be instructive to comment on the technical aspects of the

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<sup>1</sup> It should be noted that a different proof of Scarf's result has been given in Shapley (1973). In particular Shapley provides an extension of the Sperner Lemma which is used to obtain a generalized version of the Knaster-Kuratowski-Mazurkiewicz (K-K-M) theorem known in the literature as K-K-M-S. By means of the K-K-M-S theorem Shapley proves that the core of a balanced game is nonempty. Here we must note that an elegant proof of the K-K-M-S theorem was recently given by Ichiichi (1981), by using the coincidence theorem of Fan (1969).