The Core of an Economy Without Ordered Preferences

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Abstract. Core existence results are proved for exchange economies with an infinite dimensional commodity space. In particular, the commodity space may be any ordered Hausdorff linear topological space, and agents' preferences need not be transitive, complete, monotone or convex; preferences may even be interdependent. Under these assumptions a quasi equilibrium may not exist.

1. Introduction

During the last decade, contributions in consumer theory [e.g., Sonnenschein (1971), Shafer (1974) and Kim-Richter (1986)] and contributions in equilibrium theory [e.g., Mas-Colell (1974), Gale-Mas-Colell (1975), Shafer-Sonnenschein (1975), Borglin-Keiding (1976), McKenzie (1981), and Yannelis-Prabhakar (1983)] have shown that the transitivity axiom is not only a restrictive assumption but unnecessary as well. In fact, very general competitive equilibrium existence results have been obtained for finite economies where agents' preferences need not be ordered, i.e., need not be transitive or complete (therefore, need not be representable by utility functions), and may be interdependent. These existence results for the competitive equilibrium have been further generalized to economies with infinitely many commodities [see for instance Mas-Colell (1986) or Yannelis-Zame (1986) among others]. Thus, significant progress has been made on the task of establishing very general conditions for the existence of a competitive equilibrium.

The core is an alternative solution concept which has been widely used in game theory and by extension in general equilibrium analysis. It is still not known whether or not core existence results can be obtained

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