$$(5 - (q_1 + q_2))q_2 = 2.99479.$$

Hence, a value production plan is $q_1 = 1.30208$ and $q_2 = 1.19792$. To conclude, the firm with the superior information gets rewarded in the value production plan, by being assigned a higher level of production and thus higher profits ($\Pi_1 = 3.25521$ and $\Pi_2 = 2.99479$).

In the next example, the asymmetry of information comes from the cost side.

Example 9.2

Consider two firms $\{1, 2\}$ that produce a homogeneous product. The state space is $\Omega = \{a, b\}$ where each state occurs with probability $\frac{1}{2}$ and the private information of each firm is: $\mathcal{F}_2 = \{\{a\}, \{b\}\}$ and $\mathcal{F}_1 = \{\{a, b\}\}$. We denote by $q_1(\omega), q_2(\omega)$ the production in state ω of firm 1 and firm 2, respectively. The inverse demand that firms face is $p = (5 - 1.5(q_1(\omega) + q_2(\omega)))$. The marginal cost of each firm, which is measurable with respect to each firm's private information, is

$$c_2(\omega) = \begin{cases} .8 & \text{if } \omega = a \\ 1.2 & \text{if } \omega = b \end{cases},$$

$$c_1(\omega) = \begin{cases} 1 & \text{if } \omega = a \\ 1 & \text{if } \omega = b. \end{cases}.$$

A Cournot-Nash equilibrium is

$$q_1^*(a) = q_1^*(b) = .888889,$$

$$q_2^*(a) = .955556, q_2^*(b) = .822222.$$

The ex-ante expected profit for the *industry* is $\Pi_1 + \Pi_2 = 1.18519 + 1.19185 = 2.37704$.

Now suppose that the two firms collude under the common knowledge information rule. The information they use now is the trivial information and the optimum total production is 1.33333. The expected industry profits are 2.66667.

The Shapley value of the two firms (with $\lambda_1 = \lambda_2 = 1$) is

$$Sh_1 = \frac{1}{2}[2.66667 - 1.19185] + \frac{1}{2}[1.18519] = 1.33,$$

$$Sh_2 = \frac{1}{2}[2.66667 - 1.18519] + \frac{1}{2}[1.19185] = 1.33667.$$

Then, a value production will be a solution to the following problem:

$$(5 - 1.5(q_1 + q_2))q_1 - q_1 = 1.33,$$