from collusion are higher than the Cournot-Nash profits and the Shapley value of each firm captures its contribution to the total industry profits. It is important to note here that in these examples the value allocation is in the core and therefore the cartel can be viewed as *stable*, in the sense that no coalition of firms can deviate from the cartel agreement and become strictly better off.

Example 9.1

Consider two firms $\{1,2\}$ that produce a homogeneous product. The state space is $\Omega = \{a,b\}$ where each state occurs with probability $\frac{1}{2}$ and the private information of each firm is: $\mathcal{F}_1 = \{\{a\},\{b\}\}$ and $\mathcal{F}_2 = \{\{a,b\}\}$. We denote by $q_1(\omega), q_2(\omega)$ the production in state ω of firm 1 and firm 2 respectively. The inverse demand that firms face is: $p = (5 - \beta(\omega)(q_1(\omega) + q_2(\omega)))$. The marginal cost is zero for both firms. The slope β takes on the following values:

$$\beta(\omega) = \begin{cases} .8 & \text{if } \omega = a \\ 1.2 & \text{if } \omega = b \end{cases}$$

A Cournot-Nash equilibrium is

$$q_1^*(a) = 2.2916, q_1^*(b) = 1.25,$$

$$q_2^*(a) = q_2^*(b) = 1.666.$$

Notice that the production is measurable with respect to each firm's private information. The ex-ante expected profit for the *industry* is $\Pi_1 + \Pi_2 = 3.03819 + 2.77778 = 5.81597$.

Now suppose that the two firms collude under the common knowledge information rule. The information they use now is the trivial information and the optimum total production is 2.5. The expected industry profits are: 6.25. The problem that arises is how this surplus will be distributed among the two firms. Or put it in different words, what is the production that will be assigned to each firm? Without taking the information superiority of the first firm into account, both firms are identical. Hence, one solution would be just to split the profits. However, this is not a "fair solution" since firm 1 contributes more to the coalition than firm 2 does. The value production plan allocation we discussed above provides a more sensible outcome.

The Shapley value of the two firms (with $\lambda_1 = \lambda_2 = 1$) is

$$Sh_1 = \frac{1}{2}[6.25 - 2.77778] + \frac{1}{2}[3.03819] = 3.25521$$

$$Sh_2 = \frac{1}{2}[6.25 - 3.03819] + \frac{1}{2}[2.77778] = 2.99479.$$

Then, a value production will be a solution to the following problem:

$$(5 - (q_1 + q_2))q_1 = 3.25521$$