

knowledge production plan is coalitional incentive compatible. Therefore, it is of great importance if we know that there is also a way to distribute the surplus (if there is any) in a manner that the contribution of each firm is rewarded.

We now define for each Cournot game with differential information, \mathcal{C} , and for each set of weights, $\{\lambda_i : i = 1, \dots, n\}$, the associated game with side payments (I, V_λ^c) (we also refer to this as a “transferrable profits” (TP) game) as follows:

For each coalition $S \subset I$, let

$$V_\lambda^c(S) = \max_q \sum_{i \in S} \lambda_i \int \pi_i(q^S(\omega), q^{I \setminus S}(\omega)) d\mu(\omega) \quad (8.2)$$

subject to

i) for each i , q_i is $\bigwedge_{j=1}^n \mathcal{F}_j$ -measurable.

The *common knowledge value production plan* can now be defined as in definition (8.1.2), except that we replace (8.1) by (8.2) and also replace V^p by V^c .

Thus, in contrast to the private value production plan, we now require the production plan within a coalition to be based on the common knowledge information. Notice that the common knowledge value production plan is coalitional incentive compatible. However, we cannot prove a general existence theorem. In fact, if, for example, one firm has “trivial” information and the other has “full” information, the common knowledge information implies that the trivial information must be used and therefore the superadditivity condition of the function $V_\lambda^c(\cdot)$ may be violated, i.e., there can exist coalitions S, T with $S \cap T = \emptyset$ and $V_\lambda^c(S) + V_\lambda^c(T) > V_\lambda^c(S \cup T)$.¹⁴ In the proof of proposition 6.3, we present an example with two firms where the Cournot-Nash equilibrium yields higher profits than collusion under common knowledge information, which destroys the superadditivity condition. This causes problems with the existence of a common knowledge value production plan. Therefore, we cannot prove a general existence theorem of a common knowledge value production plan.

9 Examples

Below we give examples with two firms, with differential information, that collude using the common knowledge information rule and the distribution of profits is determined by a common knowledge value production plan. These examples illustrate how the common knowledge production plan is determined and also show that firms with superior information, while keeping the other characteristics of the firms (i.e., marginal cost) fixed, have higher Shapley value and higher share of the industry profits. The profits

¹⁴See Krassa and Yannelis (1996), p.177, for more details.