For each coalition $S \subset I$, let

$$V_\lambda^p(S) = \max_q \sum_{i \in S} \lambda_i \int \pi_i(q^S(\omega), q^{I \setminus S}(\omega)) d\mu(\omega) = \max_{q \in L_Q} \sum_{i \in S} \lambda_i \Pi_i(q). \quad (8.1)$$

We are now ready to define the private value production plan.

**Definition 8.1.2:** An output plan $q \in L_Q$ is said to be a *private value production plan* of the Cournot game with differential information, $C$, if there exist $\lambda_i \geq 0$ ($i = 1, \ldots, n$, which are not all equal to zero), with

$$\lambda_i \Pi_i(q) = Sh_i(V_\lambda^p), \forall i,$$

where $Sh_i(V_\lambda^p)$ is the Shapley value of firm $i$ derived from the game $(I, V_\lambda^p)$, defined in (8.1).

The above definition says that the expected profits of each firm multiplied by its weight $\lambda_i$ must be equal to its Shapley value derived from the (TP) game $(I, V_\lambda^p)$.

An immediate consequence of Definition 8.1.2 is that the private value production plan is individually rational (profits for firm $i$ are greater than of equal from the ones derived from the Cournot-Nash game). This follows immediately from the fact that the game $(I, V_\lambda^p)$ is superadditive for all weights. In addition, it is Pareto efficient.[12]

We are now ready to state the first existence result of this section.

**Theorem 8.1.1:** *Let $C = \{(Q_i, \pi_i, \mathcal{F}_i, \mu) : i = 1, \ldots, n\}$ be a Cournot game as defined in Section 3, satisfying assumptions (A.1)-(A.2).*

*Then, a private value production plan exists in $C$.*

**Proof:** This result can be proved along the lines of Krasa and Yannelis (1996), Theorem 1. □

**Remark:** One can easily show that a pooled information (where the information that is being used is the pooled information) value production plan[13] exists as well.

## 8.2 The common knowledge value production plan

We now introduce another notion of a value production plan for the Cournot game with differential information. The difference stems from the measurability restriction on the type of production plans. It is an analog of the coarse core of Yannelis (1991). We call it a common knowledge value production plan, since the information that is being used is the common knowledge information. As we saw in the previous section a common

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[12] For more details see Krasa and Yannelis (1996), p.169.

[13] Since it is not incentive compatible we will not examine it thoroughly.