

8 Distribution of profits

So far, we showed that when firms collude (under the pooled and the private information rules), industry profits are higher than the profits obtained by the Cournot-Nash game. Moreover, profits may be higher when firms collude under the common knowledge information rule. The question that remains to be answered is how are these extra profits being distributed among the firms in a way that captures the contribution of each firm to the total profits.

8.1 The private value production plan

We propose that each firm should be rewarded according to its contribution to the total profits. One way to do this is to reward each firm according to its *Shapley value*. Then a production plan will be determined, so as each firm will get its Shapley value. This production plan will be the *value production plan*.

As in the definition of the standard value allocation concept, we must first derive a transferable profit game (TP) in which each firm's profits are weighted by a factor λ_i , ($i = 1, \dots, n$), which allows for profits comparisons. In the value allocation itself no side payments are necessary.¹⁰ A game with side payments is then defined as follows:

Definition 8.1.1: A game with side payments $\Gamma = (I, V)$ consists of a finite set of agents (firms) $I = \{1, \dots, n\}$ and a superadditive, real valued function V defined on 2^I such that $V(\emptyset) = 0$. Each $S \subset I$ is called a coalition and $V(S)$ is the "worth" of the coalition S .

The Shapley value of the game Γ (Shapley 1953) is a rule that assigns to each firm i a "payoff", Sh_i , given by the formula,¹¹

$$Sh_i(V) = \sum_{\substack{S \subset I \\ S \ni \{i\}}} \frac{(|S| - 1)!(|I| - |S|)!}{|I|!} [V(S) - V(S \setminus \{i\})].$$

The Shapley value has the property that $\sum_{i \in I} Sh_i(V) = V(I)$, i.e., the Shapley value is Pareto efficient. Moreover, it is individually rational, i.e., $Sh_i \geq V(\{i\})$, $\forall i$.

We now define for each Cournot game with differential information, \mathcal{C} , and for each set of weights, $\{\lambda_i : i = 1, \dots, n\}$, the associated game with side payments (I, V_λ^p) (we also refer to this as a "transferable profits" (TP) game) as follows:

¹⁰ See Emmons and Scafuri (1985, p.60) for further discussion.

¹¹ The Shapley value measure is the sum of the expected marginal contributions a firm can make to all the coalitions of which it is a member.