and such that members of S are better off by announcing b whenever a has actually occurred. Formally, $q \in L_Q$ is said to be coalitional incentive compatible for C if it is not true that there exist coalition S, and states a, b with[8] $a \in \cap_{i \notin S} E_i(b)$, such that[9] $\pi_i(q^S(b), q^{I \setminus S}(b)) > \pi_i(q^S(a), q^{I \setminus S}(b))$, for all $i \in S$, that is, each firm in coalition S is strictly better off announcing that state b occurred rather than the true state a and firms not in S are unable to distinguish between state a and b.

It turns out that a Cournot-Nash equilibrium is incentive compatible. Also a collusion equilibrium under the common knowledge information rule is coalitional incentive compatible. However, a collusion equilibrium under the pooled information rule and under the private information rule may not be coalitional incentive compatible.

Proposition 7.1: *A Cournot-Nash equilibrium for* $C = \{(Q_i, \pi_i, \mathcal{F}_i, \mu) : i = 1, \dots, n\}$ *is incentive compatible.*

Proof: Since we are dealing with a non-cooperative concept it is appropriate to reduce the coalition S to the singleton coalition, i.e., $S = \{i\}$. Then, $-i$ denotes all the firms but i. Suppose that $q^* \in L_Q$ is a Cournot-Nash equilibrium and there exist a, b, where $a \in E_{-i}(b)$, such that

$$\pi_i(q_i^*(b), q_{-i}^*(b)) > \pi_i(q_i^*(a), q_{-i}^*(b)).$$

First, since q_{-i}^* is $\mathcal{F}_{-i}$-measurable, it is implied that $q_{-i}^*(a) = q_{-i}^*(b)$. Thus, for all $\omega \in E_i(a) \cap E_{-i}(a)$ and $t \in E_i(b) \cap E_{-i}(a)$,

$$\pi_i(q_i^*(t), q_{-i}^*(t)) > \pi_i(q_i^*(\omega), q_{-i}^*(\omega)).$$

Now consider the following production plan for firm i,

$$\tilde{q}_i(\omega) = \begin{cases} q_i^*(\omega) = q_i^*(t) & \text{if } \omega \in E_i(a) \cap E_{-i}(a) \\ q_i^*(\omega) & \text{otherwise.} \end{cases}$$

It follows that

$$\int \pi_i(\tilde{q}_i(\omega), q_{-i}^*(\omega)) d\mu > \int \pi_i(q_i^*(\omega), q_{-i}^*(\omega)) d\mu.$$

This contradicts the fact that q^* is a Nash equilibrium. □

Proposition 7.2: *A collusion equilibrium under the private information rule for* $C = \{(Q_i, \pi_i, \mathcal{F}_i, \mu) : i = 1, \dots, n\}$ *may not be coalitional incentive compatible.*

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[8] $E_i(b)$, is the event in firms' information partition that contains the realized state b.

[9] q^S and $q^{I \setminus S}$ are vectors of outputs for firms in coalition S and $I \setminus S$ respectively.