

The expected industry profits are 3.5537.

Thus, in this example the profits from collusion with common knowledge information are higher than the profits from the Cournot-Nash game. Next we present an example where the profits from the Cournot - Nash game are higher than the profits from collusion with common knowledge information.

Consider now a Cournot game with two firms $\{1, 2\}$ that produce a homogeneous output q . Uncertainty is generated by the marginal cost functions $c_i : \Omega \rightarrow \mathbb{R}_+, i = 1, 2$. Firm 1 has trivial information and its cost function is: $C_1 = c_1 q_1^2$, where $c_1 = 5$ for all $\omega \in \Omega$. Firm 2 has complete information and its cost function is: $C_2 = c_2 q_2^2$. We assume that c_2 is distributed uniformly on $[0, 10]$. The above information about the Cournot game is common knowledge. Moreover, the inverse demand function is $p(\omega) = 50 - 1.5(q_1(\omega) + q_2(\omega))$. Since firm 1 has trivial information, its production plan will be constant across all states, while firm's 2 production plan will be contingent on each realization of the random variable c_2 . Therefore, the Cournot-Nash equilibrium is

$$q_1(\omega) = 3.317, \text{ for all } \omega \in \Omega \text{ and } q_2(\omega) = \frac{22.5122}{1.5 + c_2(\omega)}.$$

The expected industry profits from the Cournot-Nash game are 174.747.

Now assume that firms collude using the common knowledge information rule. This now implies that production must be constant across all states. The production plan is

$$q_1(\omega) = q_2(\omega) = 3.125, \text{ for all } \omega \in \Omega.$$

The expected industry profits now are 156.25. \square

7 Coalitional incentive compatibility

One of the basic questions that one may ask is whether the different equilibrium notions we defined previously are coalitional incentive compatible. That is, whether a coalition of firms has an incentive to misreport the true state of nature and benefit its members. This is an important question especially for the collusion equilibrium. If a collusion equilibrium is not coalitional incentive compatible, then it is not sustainable. We define rigorously below the notion of coalitional incentive compatibility which is related to the one in Krasa-Yannelis (1994).

Definition 7.1: An output function $q \in L_Q$ is said to be *coalitional incentive compatible* if and only if the following does not hold: There exist a coalition of firms⁷ $S \subset I$ and two states a, b that members of $I \setminus S$ cannot distinguish (i.e., a and b are in the same partition for the firms in $I \setminus S$)

⁷ I is the set of all firms.