Proof: Obvious. □

However, as the following proposition indicates, we cannot compare the industry profits derived under the common knowledge rule with those derived from the Cournot-Nash game. The reason is that when firms collude using the common information, essentially they throw away some valuable information, which means lower profit. On the other hand, the joint maximization alone, gives them higher profits. In this general setting we cannot tell which effect outweighs the other.

Proposition 6.3: *The industry profits derived from the collusion under the common knowledge information rule and the ones derived from the Cournot-Nash game are not comparable.*

Proof: Consider a Cournot game with two firms $\{1,2\}$, three states of nature, i.e., $\Omega = \{a,b,c\}$ and one homogeneous output q. Each state occurs with the same probability. Each firm's private information is given by the following partition of the state space,

$$\mathcal{F}_1 = \{a,b,c\}, \mathcal{F}_2 = \{\{a\},\{b\},\{c\}\}.$$

The inverse demand function is, $p(\omega) = 5 - 1.5(q_1(\omega) + q_2(\omega))$. The cost function which is measurable with respect to each firm's private information is: For firm 1, $C_1(\omega, q_1(\omega)) = .4q_1^2$ for all $\omega \in \Omega$ and for firm 2,

$$C_2(\omega, q_2(\omega)) = \begin{cases} q_2^2 & \text{if } \omega = a \\ .4q_2^2 & \text{if } \omega = b \\ .8q_2^2 & \text{if } \omega = c. \end{cases}$$

Notice that firm 1 has trivial information, while firm 2 has complete information. The following production plan is a Cournot-Nash equilibrium,

$$q_1(\omega) = 1.00275, \text{ for all } \omega \in \Omega;$$

$$q_2(\omega) = \begin{cases} .699 & \text{if } \omega = a \\ .919 & \text{if } \omega = b \\ .759 & \text{if } \omega = c. \end{cases}$$

Observe that the production plan is also measurable with respect to each firm's private information. The expected industry profits from the Cournot-Nash game are 3.296.

Now assume that firms collude using the common knowledge information rule. Since the information that is common to both of them is the trivial information, the production plan must be constant in all states. This is,

$$q_1(\omega) = .9194, q_2(\omega) = .502, \text{ for all } \omega \in \Omega.$$