Also let $L^c_Q = L^c_{Q_1} \times \cdots \times L^c_{Q_n}$.

A *collusion equilibrium under the pooled information rule* for $C = \{(Q_i, \pi_i, \mathcal{F}_i, \mu) : 1, \ldots, n\}$ is an element $q^* \in L^p_Q$ such that

$$\Pi^p(q^*) = \max_{q \in L^p_Q} \sum_{i=1}^{n} \Pi_i(q).$$

A *collusion equilibrium under the private information rule* for $C = \{(Q_i, \pi_i, \mathcal{F}_i, \mu) : i = 1, \ldots, n\}$ is an element $q^* \in L_Q$ such that

$$\bar{\Pi}(q^*) = \max_{q \in L_Q} \sum_{i=1}^{n} \Pi_i(q).$$

A *collusion equilibrium under the common knowledge information rule* for $C = \{(Q_i, \pi_i, \mathcal{F}_i, \mu) : i = 1, \ldots, n\}$ is an element $q^* \in L^c_Q$ such that

$$\Pi^c(q^*) = \max_{q \in L^c_Q} \sum_{i=1}^{n} \Pi_i(q).$$

Next we present the second existence result of this paper.

Theorem 5.1: *Under assumptions (A.1)-(A.2), a collusion equilibrium exists for all the information rules.*

Proof: Notice that the objective function is weakly continuous and L^p_Q, L_Q and L^c_Q are non-empty and weakly compact. Therefore the maximum is attained and the argmax is the set of all equilibrium points. □

6 Comparison of profits under the three information rules

In this section, we will put the industry profits under the three different information rules in a hierarchy. It is known that under symmetric information the industry profits when firms collude are greater than or equal to the industry profits derived from the Cournot-Nash game. But what happens under differential information?

Let $\Pi^p(q^*), \bar{\Pi}(q^*)$ and $\Pi^c(q^*)$ be the value functions under the three information rules, as defined in the previous section.

Proposition 6.1: $\Pi^p(q^*) \geq \bar{\Pi}(q^*) \geq \Pi^c(q^*)$.

Proof: First observe that $\wedge_{i=1}^n \mathcal{F}_i \subseteq \mathcal{F}_i, i = 1, \ldots, n, \subseteq \vee_{i=1}^n \mathcal{F}_i$. This implies that $L^c_Q \subseteq L_Q \subseteq L^p_Q$. Since the objective functions are the same, the desired result follows. □

Let $\Pi^N(q^*)$ denote the industry profits derived from the Nash game.

Proposition 6.2: $\Pi^p(q^*) \geq \bar{\Pi}(q^*) \geq \tilde{\Pi}^N(q^*)$.