Since $\Pi(q)$ is a concave function of q_i on L_{Q_i}, it follows that φ_i is convex valued. By virtue of Berge's Maximum Theorem, it follows that φ_i is weakly u.s.c. Finally, an appeal to Weierstrass' Theorem it is guaranteed that φ_i is also a non-empty valued correspondence.

Now since each φ_i is non-empty, closed, convex valued and weakly u.s.c., it follows that likewise is $F : L_Q \to 2^{L_Q}$. Thus, the correspondence F satisfies all the conditions of the Fan-Glicksberg Fixed Point Theorem. Consequently, there exists some $q^* \in L_Q$ such that $q^* \in F(q^*)$. □

5 Collusion under different information rules

It is a well known result that a Cournot-Nash equilibrium may not be Pareto optimal. In other words, there is a surplus that has not been extracted by the firms. If the firms collude and play a cooperative game, then a Pareto optimal outcome will be reached. This problem has been examined when firms have symmetric information. However, in the presence of differential information there may be different ways for the firms to collude, depending on how they want to share their private information. Before we proceed, let's define the three different information rules that we will consider in the sequel.

Definition 5.1: A *Pooled information rule* is the one where firms share their information, i.e., the information they use is, $\mathcal{F}'_j = \vee_{i=1}^n \mathcal{F}_i, j = 1, \ldots, n$, where $\vee$ denotes the *join*.[5]

Definition 5.2: A *Private information rule* is the one where firms use their own private information, i.e., $\mathcal{F}_i, i = 1, \ldots, n$.

Definition 5.3: A *Common knowledge information rule* is the one where firms use only the information that is common to them, i.e., $\mathcal{F}'_j = \wedge_{i=1}^n \mathcal{F}_i, j = 1, \ldots, n$, where $\wedge$ denotes the *meet*.[6]

Let $L^p_{Q_i}$ denote the set of all Bochner integrable and $\vee_{i=1}^n \mathcal{F}_i$- measurable selections from the production set Q_i of firm i, i.e.,

$$\begin{aligned} L^p_{Q_i} &= \{q_i \in L_1(\mu, Y) : q_i : \Omega \to Y \text{ is } \vee_{i=1}^n \mathcal{F}_i - \text{measurable and} \\ q_i(\omega) &\in Q_i(\omega),\ \mu - a.e.\}. \end{aligned}$$

Also let $L^p_Q = L^p_{Q_1} \times \cdots \times L^p_{Q_n}$.

Let $L^c_{Q_i}$ denote the set of all Bochner integrable and $\wedge_{i=1}^n \mathcal{F}_i$− measurable selections from the production set Q_i of firm i, i.e.,

$$\begin{aligned} L^c_{Q_i} &= \{q_i \in L_1(\mu, Y) : q_i : \Omega \to Y \text{ is } \wedge_{i=1}^n \mathcal{F}_i - \text{measurable and} \\ q_i(\omega) &\in Q_i(\omega), \mu - a.e.\}. \end{aligned}$$

[5] That is the smallest σ-algebra containing all of the sub σ-algebras $\mathcal{F}_i, i = 1, \ldots, n$.

[6] That is the largest σ-algebra contained in all of the sub σ-algebras $\mathcal{F}_i, i = 1, \ldots, n$.