$Q_i : \Omega \to 2^Y$, is a non-empty, convex, weakly compact-valued and integrably bounded correspondence having an $\mathcal{F}_i$-measurable graph, i.e., $G_{Q_i} \in \mathcal{F}_i \otimes B(Y)$.

(A.2)

i) For each i, $\pi_i(\cdot,\cdot) : Q(\omega) \to \mathbb{R}$, is weakly continuous.

ii) The function π_i is concave in the i-th coordinate for all i.

iii) π_i is integrably bounded.

4 Existence of a Cournot-Nash equilibrium

We can now state the first existence result. We assume that there exists a finite or countable partition $\Lambda_i, (i = 1, \dots, n)$ of Ω, and the σ-algebra $\mathcal{F}_i$ is generated by Λ_i.

Theorem 4.1: *Let $\mathcal{C} = \{(Q_i, \pi_i, \mathcal{F}_i, \mu) : i = 1, \dots, n\}$ be a Cournot game satisfying (A.1) - (A.2). Then, there exists a Cournot-Nash equilibrium.*

Proof: For each i, define the correspondence $\varphi_i : L_{Q_{-i}} \to 2^{L_{Q_i}}$ by

$$\varphi_i(q^*_{-i}) = \{y_i \in L_{Q_i} : \Pi_i(q^*) = \max_{y_i \in L_{Q_i}} \Pi_i(q^*_{-i}, y_i)\}.$$

Also define the correspondence $F : L_Q \to 2^{L_Q}$ by

$$F(q) = \Pi_{i=1}^n \varphi_i(q^*_{-i}).$$

As in Yannelis and Rustichini (1991), we will show that the correspondence F satisfies all the hypotheses of the Fan-Glicksberg Fixed Point Theorem. It can then be easily checked that a fixed point of the correspondence F is by construction a Cournot-Nash equilibrium for $\mathcal{C}$. We will complete the proof in three steps.

I. *L_Q is non-empty, convex, weakly compact and metrizable.*

The non-emptiness of L_Q follows from the Aumann measurable selection theorem. Also, since each Q_i is non-empty, convex and weakly compact, it follows from Diestel's Theorem that each L_{Q_i} is a weakly compact subset of $L_1(\mu, Y)$. Obviously, each L_{Q_i} is convex. Furthermore, since each L_{Q_i} is a weakly compact subset of a separable Banach space $L_1(\mu, Y)$, it is also metrizable [for more details see Yannelis and Rustichini (1991), Theorem 5.1].

II. *The function Π_i is weakly continuous for each i.*

Since, by assumption, π_i is concave, weakly continuous and π_i is integrably bounded, the result follows by an application of Theorem 2.8 in Balder and Yannelis (1993).

III. *Each correspondence $\varphi_i : L_{Q_{-i}} \to 2^{L_{Q_i}}$, is non-empty, convex valued and weakly u.s.c.*