ii) $\pi_i : Q(\omega) \to \mathbb{R}$ is the *random profit function*[2] of firm i, (where $Q(\omega) = Q_1(\omega) \times \cdots \times Q_n(\omega)$);

iii) $\mathcal{F}_i$ is a sub σ-algebra of $\mathcal{F}$, which denotes the *private information* of firm i;

iv) μ is a probability measure on Ω denoting the *common prior*.

Let L_{Q_i} denote the set of all Bochner integrable and $\mathcal{F}_i$-measurable selections from the production set Q_i of firm i, i.e.,

$$L_{Q_i} = \{q_i \in L_1(\mu, Y) : q_i : \Omega \to Y \text{ is } \mathcal{F}_i - \text{measurable and } q_i(\omega) \in Q_i(\omega) \text{ and } \mu - a.e.\}.$$

Let $L_Q = L_{Q_1} \times \cdots \times L_{Q_n}$. Given a Cournot game, a production plan for firm i is an element $q_i \in L_{Q_i}$.

The *ex-ante expected profit function*[3] of firm i, $\Pi_i : L_Q \to \mathbb{R}$ is defined as[4]

$$\Pi_i(q_i, q_{-i}) = \int_{\omega\in\Omega} \pi_i(q_i(\omega), q_{-i}(\omega))d\mu(\omega).$$

A *Cournot-Nash equilibrium* for $C = \{(Q_i, \pi_i, \mathcal{F}_i, \mu) : i = 1, \ldots, n\}$ is an element $q^* \in L_Q$ such that for all i,

$$\Pi_i(q^*) = \max_{y_i \in L_{Q_i}} \Pi_i(q^*_{-i}, y_i).$$

We can now state the assumptions needed to prove the existence of a Cournot-Nash equilibrium.

(A.1)

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[2] *If $p(\omega) : Q(\omega) \to R$ is the inverse demand function and $C_i : Q_i(\omega) \to R$ is the cost function of firm i, then $\pi_i(q(\omega)) = p(q(\omega))q_i(\omega) - C_i(q_i(\omega))$. We could have allowed the payoff function π to depend also on the state of nature ω. The results of the paper remain valid.*

[3] The entire analysis would go through if instead of the ex-ante profit function we used the interim one. That is, the *conditional (interim) expected profit function* of firm i $\Pi_i(\cdot,\cdot) : L_{Q_i} \times Q_{-i}(\omega) \to R$ is defined as

$$\Pi_i(q_i, \bar{q}_{-i}) = \int_{\omega' \in E_i(\omega)} \pi_i(\omega', q_i, \bar{q}_{-i}(\omega'))k_i(\omega'|E_i(\omega))d\mu(\omega'),$$

where

$$k_i(\omega'|E_i(\omega)) = \begin{cases} 0 & \text{if } \omega' \notin E_i(\omega) \\ \frac{q_i(\omega')}{\int_{\tilde{\omega}\in E_i(\omega)} q_i(\tilde{\omega})d\mu(\tilde{\omega})} & \text{if } \omega' \in E_i(\omega) \end{cases}$$

is the prior of agent i, (where k_i is a Radon-Nikodym derivative such that $\int k_i(\omega)d\mu(\omega) = 1$ and $E_i(\omega)$ denotes the event in firm i's partition which contains the realized state of nature).

[4] For simplicity we assume that the profit function does not depend on Ω. As we mentioned above all the results of the paper remain valid.