of Aumann tells us that if $(\Omega, \mathcal{F}, \mu)$ is a complete finite measure space, X is a separable metric space and $\phi : \Omega \to 2^X$ is a non-empty valued correspondence having a measurable graph, then $\phi(\cdot)$ admits a *measurable selection*, i.e., there exists a measurable function $f : \Omega \to X$ such that $f(\omega) \in \phi(\omega), \mu - a.e.$

Let $(\Omega, \mathcal{F}, \mu)$ be a finite measure space and X be a Banach space. Following Diestel-Uhl (1977), the function $f : \Omega \to X$ is called *simple* if there exist $x_1, x_2, ..., x_n$ in X and $\alpha_1, \alpha_2, ..., \alpha_n$ in $\mathcal{F}$ such that $\sum_{i=1}^n x_i \chi_{\alpha_i}$ where $\chi_{\alpha_i}(\omega) = 1$ if $\omega \in \alpha_i$ and $\chi_{\alpha_i}(\omega) = 0$ if $\omega \notin \alpha_i$. A function $f : \Omega \to X$ is said to be *μ-measurable* if there exists a sequence of simple function $f_n : \Omega \to X$ such that $\lim_{n\to\infty} \|f_n(\omega) - f(\omega)\| = 0$ for almost all $\omega \in \Omega$. A *μ-measurable* function $f : \Omega \to X$ is said to be *Bochner integrable* if there exists a sequence of simple functions $\{f_n : n = 1, 2, ...\}$ such that

$$\lim_{n\to\infty} \int_\Omega \|f_n(\omega) - f(\omega)\| d\mu(\omega) = 0.$$

In this case we define for each $E \in \mathcal{F}$ the integral to be

$$\int_E f(\omega) d\mu(\omega) = \lim_{n\to\infty} \int_E f_n(\omega) d\mu(\omega).$$

It can be shown [see Diestel-Uhl (1977), Theorem 2, p.45] that if $f : \Omega \to X$ is a μ- measurable function then f is Bochner integrable if and only if $\int_\Omega \|f(\omega)\| d\mu(\omega) < \infty$.

For $1 \le p < \infty$, we denote by $L_p(\mu, X)$ the space of equivalence classes of X-valued Bochner integrable functions $x : \Omega \to X$ normed by

$$\|x\|_p = \left(\int_\Omega \|x(\omega)\|^p d\mu(\omega)\right)^{\frac{1}{p}}.$$

It is a standard result that normed by the functional $\|\cdot\|_p$ above, $L_p(\mu, X)$ becomes a Banach space [see Diestel-Uhl (1977), p.50].

3 The Cournot game with differential information

We assume that there are n firms, $\{i = 1, \dots, n\}$, that produce an output $q = \{q_1, \dots, q_n\}$. The subscript $-i$ will be used to denote all firms other than firm i. Let $(\Omega, \mathcal{F}, \mu)$ be a complete probability measure space. We interpret Ω as the states of nature of the world and assume that it is large enough to include all events that we consider to be interesting. As usual $\mathcal{F}$ denotes the σ-algebra of events and μ is a common probability measure. Let Y be a separable Banach space denoting the production space.

Definition 3.1: A *Cournot game with differential information* is a set $\mathcal{C} = \{(Q_i, \pi_i, \mathcal{F}_i, \mu) : i = 1, \dots, n\}$, where

i) $Q_i : \Omega \to 2^Y$ is the *random production set* of firm i;