has to be northeast (else, 4 $\wedge$ 2) (see Figure 15 (a)). Since 2 does not see 4′, 2′ sees 4′ to its north, and 2′ sees 2 to its northwest, Observation 2 implies that a N dent separates 2 and 4′. By a similar argument, a S dent separates 4 from 2′. This establishes that there exist 4 different dent orientations, a contradiction.

Case 2: Let 4 be on the same side of *Cons* (1,3) as 3.
Then the staircase from 4 to 4′ on the 1 dent line, and hence to 4″ on the 3 dent line, has to be northwest (else, 4 $\wedge$ 2) (see Figure 15 (b)) Since 2 does not see 4′, 2′ sees 4′ to its north, and 2′ sees 2 to its northwest, Observation 2 implies that a N dent separates 2 and 4′. By a similar argument, a S dent separates 4 from 2″. This establishes the existence of four different dent orientations, a contradiction.

Q.E.D..

Reckhow and Culberson [19] show that if P has only three dent orientations, the number of sources and the number of sinks are both $O(n)$. For a source v and sink u, it is easy to figure out in $O(n)$ time if $u \equiv v$ [5]. Thus, in $O(n^3)$ time, we can list the sources seen by each sink. Now, for each pair of sources, we can figure out in $O(n)$ time if they see each other indirectly, thus constructing the sink graph H in $O(n^3)$ time (H has $O(n^2)$ edges). Gavril's algorithm [8] now gives, in $O(n^3)$ time, the minimum clique cover of H, implying that the the star cover of P can be obtained in $O(n^3)$ time.

9. Conclusions

We have shown that the minimum clique cover of the star graph of a simple orthogonal polygon corresponds exactly to a minimum star cover of the polygon. We have further shown that this graph is weakly triangulated. By Hayward's Theorem, weakly triangulated graphs are perfect [11]. This implies the following duality relationship: the minimum number of star polygons needed to cover an orthogonal polygon is equal to the maximum number of points in the polygon, no two of which can be contained together in a single star polygon. Further, The Ellipsoid method of Grötschel, Lov´asz and Schrijver provides a polynomial algorithm to find a minimum clique cover of perfect graphs, and hence to cover such polygons with a minimum number star polygons.

In the case where the polygon has only three dent orientations, we have shown that the star graph is triangulated. By Gavril's algorithm, we can find a minimum star cover for such polygons in $O(n^3)$ time.