

Since 6 is not in $Cons(2, 3)$, and $6 \wedge 2$ and $6 \wedge 3$, Property 4 implies that there is a single staircase path K from 6 to the 2 and 3 dent lines of $Cons(2, 3)$, and passing through $Cons(2, 3)$. Since the 1 dent line of $Cons(1, 2)$ is in $Cons(2, 3)$, (see Figure 13 (a)), K intersects the 1 dent line of $Cons(1, 2)$, implying that $6 \wedge 1$. But $\langle 6, 1 \rangle$ is a cycle edge of H^c , a contradiction.

Q.E.D.

8. An $O(n^3)$ Algorithm for Polygons with Three Dent Orientations

In this section, we show that if P has only three dent orientations, then the star graph H is triangulated [9, 15]. This would give us an $O(n^3)$ algorithm for finding the star cover for P .

Definition 5. [9, 15] A graph is said to be triangulated (or chordal) if it contains no induced cycles of length greater than three.

In general, the star graph H is not triangulated: there exist induced 4 cycles in H (see Figure 14). For the case where P has only three dent orientations, H is triangulated, as the following theorem shows.

Theorem 5. The star graph H of an orthogonal polygon with at most three dent orientations is triangulated.

Proof: By Lemma 12, we have that H does not contain induced cycles of length greater than four. It now suffices to show that H cannot have an induced 4 cycle.

Assume to the contrary that $\langle 1, 2 \rangle, \langle 2, 3 \rangle, \langle 3, 4 \rangle$ and $\langle 4, 1 \rangle$ is an induced 4 cycle in H . Since $\langle 1, 3 \rangle$ is not present, $1 \star 3$. Consider $Cons(1, 3)$. Neither 2 nor 4 is in $Cons(1, 3)$, else by Lemma 9, $2 \wedge 4$. This also implies that $Cons(1, 3)$ is not of type III.

If $Cons(1, 3)$ is of type II, then by Lemma 11, $2 \wedge 4$, a contradiction.

Thus, $Cons(1, 3)$ is a type I constriction. Without loss of generality, let the two dent lines be vertical and let the 1 dent line be to the west of the 3 dent line (see Figure 15 (a)). By Property 2, we have established the presence of an E dent and a W dent.

Without loss of generality, let 2 be on the same side of $Cons(1, 3)$ as 1. By Property 4, there is a staircase L (southeast, say) from 2 to $2''$ on the 3 dent line, intersecting the 1 dent line at $2'$. 2 sees every point below $2'$ on the 1 dent line and every point below $2''$ on the 3 dent line.

Case 1: Let 4 be on the same side of $Cons(1, 3)$ as 1.

Then, every staircase from 4 to $4''$ on the 3 dent line, and hence to $4'$ on the 1 dent line,