

Now, let $Cons(1, 3)$ be of type I, and let the two dent lines be vertical (the other case is similar). Further, let 2 be on the same side of $Cons(1, 3)$ as 1. By the proof of Property 4, there is a staircase L (southeast, say) to the 3 dent line, meeting the 1 and 3 dent lines at $2'$ and $2''$, respectively. Therefore, 2 sees every point below $2'$ and $2''$ on the 1 and 3 dent lines, respectively (see Figure 10). To prevent chords, R needs to meet the 1 and 3 dent lines (at z and y) above $2'$ and $2''$.

Case 1: $2' \neq y$ (see Figure 11).

$2''$ sees y to its north, and sees $2'$ to its northwest. By Observation 2, there is a N dent D separating $2'$ and y . We have $2' \in B_l(D)$ and $y \in B_r(D)$. $2'' \equiv 2'$ and $2'' \equiv y$, implying that $2'' \in A(D)$. Thus, L crosses \vec{D} at some point, say l' . Since z is to the north of $2'$, we have that $z \in B_l(D)$. Thus, R is a sequence of 1-bend paths from a point in $B_l(D)$ to a point in $B_r(D)$. This implies that R cross \vec{D} at x to the east of l' . Hence, $2 \equiv x$, implying that $2 \wedge i$, for some $i \in \{4, \dots, n\}$, a contradiction.

Case 2: $2' \equiv y$ (see Figure 12).

If the staircase from $2'$ to y is southeast, then the concatenation of this staircase with the southeast staircase from 2 to $2'$ gives us a southeast staircase from 2 to y , implying that $2 \wedge i$, for some $i \in \{4, \dots, n\}$, a contradiction. Therefore, the staircase from $2'$ to y is northeast. Let R' be such a staircase from $2'$ to y such that no other such staircase lies entirely to its north. Thus, some horizontal edge of R' touches a polygon edge. R', L and the 3 dent line between y and $2''$ together form an OCP, S . We now assert that R has a point in S . Assume to the contrary that R does not have a point in S . Then $z \notin S$. This means that z is above R' on the 1 dent line. By our choice of R' , y and z are now in different connected components of the portion of P that is bounded by R' and the two dent lines (see Figure 11). Since this portion of P borders S at R' , and R is a connected path, R must have a point in S .

Since R is a sequence of 1-bend paths, and S is in $Cons(1, 3)$, one of the points of R in S is either $i \in \{4, \dots, n\}$ or a bend point x such that x sees some $j \in \{5, \dots, n\}$ (every bend point of R that is in $Cons(1, 3)$ sees two points in $\{4, \dots, n\}$). If $i \in S$, then the vertical line from i to L exists in S (S is an OCP), thus establishing a chord between 2 and i . If there is no $i \in \{4, \dots, n\}$ in S , then we have a bend point, $x \in S$, such that $x \equiv j$, for some $j \in \{5, \dots, n\}$. By our choice of S , j is above R' in $Cons(1, 3)$. Since R' is a northeast staircase from $2'$ to y , the staircase from j to x can only travel southwest, southeast or northeast. If it is southwest or southeast, then $2 \wedge j$ by using a vertical line from x to L . If it is northeast, then $3 \wedge j$ by using a horizontal line from x to the 3 dent line, implying that $3 \wedge j$, a contradiction.

Q.E.D.