

Theorem 3. (Hayward) Weakly triangulated graphs are perfect.

We now state our main result, the proof of which is contained in the next two sections.

Theorem 4. The star graph of an orthogonal polygon P is weakly triangulated.

Theorem 4, together with Hayward's theorem, provides us with the following duality relationship:

Corollary 2. (The Duality Relationship) The minimum number of star polygons needed to cover an orthogonal polygon P is equal to the maximum number of points of P , no two of which see each other by 1-bend paths.

We can now use any algorithm that would cover the vertices of a weakly triangulated graph with a minimum number of cliques to cover an orthogonal polygon with a minimum number of star polygons. The best known algorithm to find minimum clique cover of a perfect graph, which is due to Grötschel, Lovász and Schrijver, is based on the Ellipsoid method, and it runs in polynomial time [10]. Reckhow and Culberson [5] provide a polynomial algorithm to compute the instance of the set covering problem in Section 2. The star graph can easily be obtained from this. Thus, there is a polynomial algorithm for finding a minimum star cover for P .

6. Induced Cycles of the Star Graph

In this section, we establish one part of the proof of Theorem 4, namely that the star graph contains no induced cycles of length greater than four.

Lemma 12. The star graph H does not contain an induced cycle of length greater than four.

Proof: Assume to the contrary that $C = (V', E')$ is such an induced cycle, $|V'| > 4$. For convenience, let $V' = \{1, 2, \dots, n\}$, $n > 4$, and let $\langle i, i + 1 \pmod n \rangle \in E'$. By assumption, edge $\langle 1, 3 \rangle \notin E$, implying that $1 \wedge 3$. Hence, we have the constriction $Cons(1, 3)$.

We now assert that 2 cannot be in $Cons(1, 3)$. Assume to the contrary that $2 \in Cons(1, 3)$. $R = C - \{2\}$ is a sequence of 1-bend paths connecting 1 and 3, and hence by Lemma 10, $2 \wedge i$, for some $i \in \{4, \dots, n\}$, thus establishing a chord.

This also implies that $Cons(1, 3)$ is not of type III. Further, by Lemma 11, if $Cons(1, 3)$ is of type II, $2 \wedge i$, for some $i \in \{4, \dots, n\}$, establishing a contradiction.