

$\vec{D}$  at some point, say  $l'$ . By Property 4, there is a path  $M$  from  $s'$  to  $s''$  on the  $q$  dent line that is either a staircase or a 1-bend path with  $s$  as the bend point. Since  $s'' \in B_l(D)$ ,  $M$  crosses  $\vec{D}$  at  $m'$ , such that  $m'$  is to the west of  $l'$ . Hence,  $r \equiv m'$ , implying that  $r \wedge s$ .

Q.E.D.

**Lemma 10.** Let  $r \in \text{Cons}(p, q)$ , such that  $r \wedge p$  and  $r \wedge q$ . Let  $R = (p r_1, r_1 r_2, \dots, r_{k-1} r_k, r_k q)$  be a sequence of 1-bend paths. Then,  $\exists i \in \{1, \dots, k\}$  such that  $r \wedge r_i$ .

*Proof:* By a proof very similar to that of Lemma 9, we can establish that  $r$  sees some point, say  $m'$ , of  $R$  such that  $m' \in \text{Cons}(p, q)$ . Since  $R$  is a sequence of 1-bend paths, and  $m' \in \text{Cons}(p, q)$ , there exists  $r_i \in \{r_1, \dots, r_k\}$ , such that  $r_i \equiv m'$ , implying that  $r \wedge r_i$ .

Q.E.D.

**Lemma 11.** Let  $\text{Cons}(p, q)$  be of type II. Let  $r \in \text{Cons}(p, q)$ , such that  $r \wedge p$  and  $r \wedge q$ . Let  $R = (p r_1, r_1 r_2, \dots, r_{k-1} r_k, r_k q)$  be a sequence of 1-bend paths. Then,  $\exists i \in \{1, \dots, k\}$  such that  $r \wedge r_i$ .

*Proof:* Let  $r$  be on the same side of  $\text{Cons}(p, q)$  as  $p$  (the other case is symmetrical). Without loss of generality, assume that the  $p$  dent line is vertical and the  $q$  dent line is horizontal. By Property 4, there is a staircase path  $L$  (southeast, say) from  $r$  to  $r''$  on the  $q$  dent line that meets the  $p$  dent line at  $r'$ . Clearly,  $r$  sees every point of the  $p$  dent line below  $r'$  and every point of the  $q$  dent line to the right of  $r''$  (see Figure 9). For  $r$  not to see some point of  $R$  inside  $\text{Cons}(p, q)$ ,  $R$  has to meet the  $p$  dent line above  $r'$ , and the  $q$  dent line to the left of  $r''$ . Thus,  $R$  meets  $L$  inside  $\text{Cons}(p, q)$ , implying that  $r \wedge r_i$ , for some  $r_i \in \{r_1, \dots, r_k\}$ .

Q.E.D.

## 5. Weakly Triangulated Graphs and the Star Graph

In this section, we assert that the star graph introduced in the previous section belongs to a special class of graphs called perfect graphs [9, 15]. In a perfect graph  $G$ , the size of a minimum clique cover of every induced subgraph  $G'$  is equal to the size of a maximum independent set of  $G'$ . A minimum clique cover of a perfect graph can be found in polynomial time [10]. We first need the following definition.

**Definition 4.** A graph  $G$  is *weakly triangulated* [11] if neither  $G$  nor  $G^c$ , the complement of  $G$  contain induced cycles of length greater than four.