Property 4. Let $p \not\wedge q$, and let $r \wedge p$ and $r \wedge q$. Then, (1) If $r \in Cons(p, q)$, there is a 1-bend path from a point on the p dent line to a point on the q dent line with r as bend point, (2) If $r \notin Cons(p, q)$, r has a single staircase path that meets the p and q dent lines.

Proof: If $r \in Cons(p, q)$, we have that $r \not\equiv p$ and $r \not\equiv q$, implying that there are staircase paths from r to points on the p and q dent lines. If $r \notin Cons(p, q)$, assume, without loss of generality, that r is on the same side of $Cons(p, q)$ as p. Now, clearly, $r \not\equiv q$, implying that there is a staircase path L from r to the q dent line. L has to intersect the p dent line, thus establishing a staircase to both the p and q dent lines.

Q.E.D.

We now define three types of constrictions. The other types of constrictions do not play a part in this paper.

Type I. In a type I constriction, the p and q dent lines are parallel and there exist points r_1 and r_2 on the p and q dent lines respectively such that $r_1 \equiv r_2$.

Type II. In a type II constriction, the p and q dent lines are orthogonal and there exist points r_1 and r_2 on the p and q dent lines respectively such that $r_1 \equiv r_2$.

Type III. In a type III constriction, $Cons(p, q)$ is not of type I or II, and there exist points r_1 and r_2 on the p and q dent lines respectively such that $r_1 \wedge r_2$. Thus, if $r \wedge p$ and $r \wedge q$, then $r \in Cons(p, q)$.

The next four results will be used repeatedly in the following three sections. In what follows, $p, q \in P$, such that $p \not\wedge q$.

Lemma 8. If $\exists\ r \in P$, such that $r \wedge p$ and $r \wedge q$, then $Cons(p, q)$ is of type I, II or III.

Proof: Trivial.

Q.E.D.

Lemma 9. Let $r \in Cons(p, q)$, such that $r \wedge p$ and $r \wedge q$. Further, let $s \wedge p$ and $s \wedge q$. Then $r \wedge s$.

Proof: By Property 4, there is a 1-bend path L from r' on the p dent line to r'' on the q dent line with r as bend point. Without loss of generality, let the p dent line be vertical, and let the staircase path from r to r' be southwest (the other cases are handled similarly). Thus, r sees every point below r' on the p dent line (see Figure 8).

Assume to the contrary that $r \not\wedge s$. This implies that s does not see r' or any point below r' on the p dent line. Therefore, let s see a point s' on the p dent line that is above r'. Clearly, $r \not\equiv s'$. Since r' sees s' to its north, and r' sees r to its northeast, Observation 2 implies that there is a N dent D separating r and s'. We have $s' \in B_l(D)$ and $r \in B_r(D)$ (see Figure 8). $r' \equiv s'$ and $r' \equiv r$, implying that $r' \in A(D)$. Thus, L crosses