Lemma 7. There is a unique Q_i for which $v(p)$, $v(q)$ and Q_i form a connected polygon.

Proof: Clearly, $Q_i \cap Q_j = \emptyset$ for $i \neq j$. Since P is connected, this implies that there exists at least one i such that $P_i = Q_i \cup [v(p) \cup v(q)]$ is connected. Now, let there be $i, j, i \neq j$ such that P_i and P_j are connected. Then, $[v(p) \cup v(q)] \cup [Q_i \cup Q_j]$ bounds a region S that is disjoint from it. Since P is simple, every point in S is in P. This implies that Q_i and Q_j were not connected components of $P - [v(p) \cup v(q)]$, a contradiction.

Q.E.D.

Definition 3. Q_i is called the *constriction* between p and q, and is denoted by $Cons(p, q)$. (see Figure 7)

We now state four important properties concerning $Cons(p, q)$.

Property 1. The boundary between $Cons(p, q)$ and $v(p)$ (resp., $v(q)$) is a single line segment (horizontal or vertical), called the p dent line (resp., the q dent line).

Proof: Consider the orthogonal polygon $P - v(p)$. By Definition 3, $Cons(p, q)$ and $v(q)$ lie in the same connected component S of $P - v(p)$. The boundary of S with $v(p)$ is the same as the boundary of $Cons(p, q)$ and $v(p)$, which, by Lemma 1, is a single line segment (horizontal or vertical). Similarly, the boundary of $v(q)$ and $Cons(p, q)$ is a single line segment (horizontal or vertical).

Q.E.D.

Property 2. Let the p dent line of $Cons(p, q)$ be vertical, and let $Cons(p, q)$ lie to its east (west). Let r' be a point on the p dent line, and let r be any point to the east (resp., west) of r', such that $r \equiv r'$. Then, an **E** dent (resp., **W** dent) separates p from r. (Similar statements can be made about the other two dent orientations).

Proof: Since p sees r', and does not see r at an infinitismal distance to the east of r, no staircase from p to r' can travel to the northeast or southeast (see Figure). Thus, r' sees p to its northeast or southeast, and sees r to its east, and $p \not\equiv r$. By Observation 2, there is an **E** dent separating p and r. The proof for the other part of Property 2 is similar.

Q.E.D.

Property 3. Let R be a path in P connecting p and q. Further, let $p \not\ast q$. Then, there is a connected subpath of R that joins a point on the p dent line and a point on the q dent line of $Cons(p, q)$ and is completely contained in $Cons(p, q)$.

Proof: This follows trivially from the definition of $Cons(p, q)$.

Q.E.D.