see each other.

Lemma 4. [5] Two regions indirectly see each other if and only if there exists a sink u in P that sees both.

Clearly, two points of P see each other indirectly if and only if their visibility polygons have at least one point in common. The following two results show that if two points indirectly see each other, then the intersection of their visibility polygons is a simply connected region. Further, if three points indirectly see each other, then there is at least one point that sees all three.

Lemma 5. Let $p, q \in P$ such that $v(p) \cap v(q) \neq \emptyset$. Then, $v(p) \cap v(q)$ is a simply connected polygon.

Proof: Assume to the contrary that $v(p) \cap v(q)$ is not simply connected. Let $W = v(q) - [Ep \cap Eq]$. Since $v(p) \cap v(q)$ is not simply connected, the boundary between $v(q)$ and W is not simply connected. As in the proof of Lemma 1, $v(q) \cup W$, which is $v(p) \cup v(q)$, bounds a region S that is disjoint from $v(q) \cup W$, or $v(p) \cup v(q)$. Every point of S is in P, as P is a simple polygon. Therefore, the boundary of S is composed entirely of the boundary of $v(p) \cup v(q)$, not including edges of P. Let R be the connected component of $P - v(p)$ that contains S. By Lemma 1, the boundary of $v(p)$ with R is a single line segment, implying that the boundary of $v(p)$ with S is a single line segment. Similarly, the boundary of $v(q)$ with S is a single line segment. Since at least four orthogonal line segments are required to enclose a region, we obtain a contradiction.

Q.E.D.

Lemma 6. Let $p, q, r \in P$ such that $v(p) \cap v(q) \neq \emptyset$, $v(q) \cap v(r) \neq \emptyset$ and $v(r) \cap v(p) \neq \emptyset$. Then, $v(p) \cap v(q) \cap v(r) \neq \emptyset$.

Proof: Assume to the contrary that $v(p) \cap v(q) \cap v(r) = \emptyset$. Since $v(p) \cap v(q) \neq \emptyset$, $v(q) \cap v(r) \neq \emptyset$ and $v(r) \cap v(p) \neq \emptyset$, we can show, by a proof similar to that of Lemma 5, that $v(p) \cup v(q) \cup v(r)$ bounds a region, say S, that is disjoint with $v(p) \cup v(q) \cup v(r)$. We can now show, again, by a proof similar to that of Lemma 5, that this is impossible, because three orthogonal lines cannot bound a region. This establishes that $v(p) \cap v(q) \cap v(r) \neq \emptyset$.

Q.E.D.