

If c lies to the south of b , then Q is to the east of bc (see Figure 4). As before, p lies to the east of bc . Clearly, the staircase path from p to b must pass through Q , a contradiction.

Q.E.D.

2.3. Dent Lines and Zones

For each dent edge D , we construct a *dent line* \vec{D} by extending D in both directions until it meets the boundary of P . The orientation of \vec{D} is the same as the orientation of D . \vec{D} divides P into three *zones*. Two of these zones (labeled $B_l(\vec{D})$ and $B_r(\vec{D})$ in Figure 5) are said to be *below* the dent, and are the two connected components of P below \vec{D} . The third zone, $A(\vec{D})$ is *above* \vec{D} , and is the connected component of P above \vec{D} . For any $p \in B_l(D), q \in B_r(D), p \neq q$. Thus, if $p \equiv r$ and $q \equiv r$, then $r \in A(D)$. Also, if $p \neq q$, then there is a dent D , such that $p \in B_l(D), q \in B_r(D)$, or vice versa. We say that D *separates* p and q , and D itself is called a separating dent between p and q . Note that when there are several separating dents, we will confine our attention to any one separating dent. The following fact is from [18].

Observation 2. Let L be a southeast staircase path from p to q and M be a northeast staircase path from p to r , where p, q and r are points in P . If $q \neq r$, then an E dent D separates q and r . (Similar statements can be made about the other three dent orientations.)

2.4. The Region DAG

The set of all dent lines of P subdivides P into *regions*. Reckhow and Culberson [19] construct a region DAG (directed acyclic graph) as follows: The vertices of the region DAG correspond to the regions, and there is an arc from u to v if u and v share a common border \vec{D} and u is *below* \vec{D} . A *source* is a region of zero in-degree in the region DAG. Similarly, a *sink* is a region of zero out-degree in the region DAG. (see Figure 6). Let V denote the set of all sources of P and U denote the set of all sinks of P .

Definition 2. Let u and v be any two regions of the region DAG. Region u is said to *see* region v ($u \equiv v$) if some OCP includes both u and v .

Observation 3. [18] Let u and v be any two regions of the region DAG, and let q_u and q_v be any two points in u and v respectively. Then, $q_u \equiv q_v$ iff $u \equiv v$.