

We say that a staircase path from  $p$  to  $q$  goes southwest if, in traversing it from  $p$  to  $q$ , we go west on all the horizontal segments and south on all vertical segments. Thus, staircase paths between  $p$  and  $q$  can be of four possible orientations: northeast, northwest, southeast, southwest.

We now define a star polygon.

**Definition 1.** A *star polygon (SP)*  $P'$  is an OP such that  $\exists p \in P'$  with the property that  $\forall q \in P', p \equiv q$ .

A *maximal SP* in  $P$  is an SP contained in  $P$ , but not contained in any other SP contained in  $P$ . The *visibility polygon*,  $v(p)$ , of  $p \in P$  is the set of all points  $q \in P$ , such that  $p \equiv q$ .

**Lemma 1.** The boundary between  $v(p)$  and any connected component of  $P - v(p)$  is a single line segment (horizontal or vertical).

*Proof:* The proof is in two parts. We first show that the boundary between  $v(p)$  and any connected component  $Q$  of  $P - v(p)$  is connected. Assume to the contrary that the boundary is disconnected. Then, there is a region  $S$  that is bounded by  $[v(p) \cup Q]$ , and disjoint from  $[v(p) \cup Q]$ . Since  $P$  is simple,  $S$  is contained in  $P$ . For the same reason, the boundary of  $S$  with  $v(p)$  or  $Q$  cannot use a polygon edge. By definition no point of  $S$  is in  $v(p)$ . Then,  $S \cup Q$  is connected and  $[S \cup Q] \in P - v(p)$ , implying that  $Q$  is not a connected component of  $P - v(p)$ , a contradiction.

We now show that the boundary between  $v(p)$  and  $Q$  is a single line segment. Assume to the contrary that the boundary contains adjacent line segments  $ab$ , that is horizontal, and  $bc$ , that is vertical. Assume without loss of generality, that  $v(p)$  lies to the south of  $ab$ . If  $p$  lies to the south of  $ab$ , then the staircase paths from  $p$  to any point on  $ab$  goes northeast or northwest. Such a staircase path can be extended vertically upwards to see points in  $Q$ , implying that  $ab$  is not part of the boundary. Thus,  $p$  lies to the north of  $ab$ .

If  $c$  lies to the north of  $b$  then  $Q$  is to the west of  $bc$  (see Figure 3). By an argument similar to the one for  $ab$ , this implies that  $p$  is to the west of  $bc$ . Now, the staircase paths from  $p$  to  $a$  and  $c$ , together with  $ab$  and  $bc$  form an OCP, which must have a non-empty intersection with  $Q$ . This implies, by Observation 1, that  $p$  sees points in  $Q$ , a contradiction.