PROOF: By definition. □

THEOREM 2.1.2 : In a two-person exchange economy, $C(\mathcal{E}) = [WP(\mathcal{E}) \cap IR(\mathcal{E})]$.

PROOF: Let $x \in \boldsymbol{C}(\mathcal{E})$. From the previous theorem, we know that $x \in \boldsymbol{WP}(\mathcal{E})$. Now suppose that $x \notin \boldsymbol{IR}(\mathcal{E})$. Then there is an agent i such that $e_i \succ_i x_i$. Then $\{i\}$ is a coalition who can block the allocation x, which is a contradiction. To prove the reverse direction, choose $x \in WP(\mathcal{E}) \cap IR(\mathcal{E})$ and suppose $x \notin \boldsymbol{C}(\mathcal{E})$. Then there is a coalition $S \subset I$ and $(x'_i)_{i \in S} \in \Pi_{i \in S} X_i$ such that $\sum_{i \in S} x'_i = \sum_{i \in S} e_i$ and $x'_i \succ_i x_i, \forall i \in S$. If $S = \{i\}$, x is not individually rational, a contradiction. If $S = I$, x is not weakly Pareto optimal, a contradiction. □

THEOREM 2.1.3 : $\boldsymbol{W}(\mathcal{E}) \subset \boldsymbol{C}(\mathcal{E})$.

PROOF: Choose $x \in \boldsymbol{W}(\mathcal{E})$ and corresponding prices p. Suppose $x \notin \boldsymbol{C}(\mathcal{E})$. Then there is a coalition $S \subset I$ and $(x'_i)_{i \in S} \in \Pi_{i \in S} X_i$ such that $\sum_{i \in S} x'_i = \sum_{i \in S} e_i$ and $x'_i \succ_i x_i, \forall i \in S$. Then since x is Walrasian equilibrium allocation, $p \cdot x'_i > p \cdot e_i, \forall i \in S$. Thus, $p \cdot \sum_{i \in S} x'_i > p \cdot \sum_{i \in S} e_i$, which is a contradiction. □

N. B. First Welfare Theorem II is a corollary of these two theorems.

THEOREM 2.1.4 : $C(\mathcal{E})$ is nonempty and compact.

THEOREM 2.1.5 : Let $C(\mathcal{E}^r)$ be the core of r-th replica economy. Define $C^r(\mathcal{E}) = \{x \in X : x_i = x^*_{ij}, \forall i, \text{where } x^* \in C(\mathcal{E}^r)\}$. Then $C^{r+1}(\mathcal{E}) \subset C^r(\mathcal{E}), \forall r \in \mathbf{N}$.

DEFINITION 2.1.2 : The set of **Edgeworth equilibria** is the set $E(\mathcal{E}) = \bigcap_{r=1}^{\infty} C^r(\mathcal{E})$.

LEMMA 2.1.6 : If any finite intersection in a family of nonempty compact sets is nonempty, then the intersection of the whole family is nonempty.

THEOREM 2.1.7 : $E(\mathcal{E}) \neq \emptyset$.

THEOREM 2.1.8 (Edgeworth Conjecture) : $E(\mathcal{E}) = W^*(\mathcal{E})$, where $W^*(\mathcal{E})$ is the set of Walrasian equilibrium allocation in $\mathcal{E}$.