N. B. The assumption in the theorem is called *gross substitutability*. Under the assumption of gross substitutability, the equilibrium is unique.

1.7 Optimality of Walrasian Equilibrium

1.7.1 Definitions

DEFINITION 1.7.1 : A $(p, x) \in \Delta \times X$ is a **Walrasian equilibrium** if x is feasible and $x'_i \succ_i x_i \Rightarrow p \cdot x'_i > p \cdot e_i \geq p \cdot x_i, \forall i \in I$.

DEFINITION 1.7.2 : A $(p, x) \in \Delta \times X$ is a **quasi-equilibrium** if x is feasible and $x'_i \succeq_i x_i \Rightarrow p \cdot x'_i \geq p \cdot e_i, \forall i \in I$

DEFINITION 1.7.3 : A (p, x) is a **proper quasi-equilibrium** if it is a quasi-equilibrium and $p \cdot \sum_i x_i \neq p \cdot \sum_i x'_i$ for some feasible allocation x'.

DEFINITION 1.7.4 : A feasible allocation $x \in X$ is said to be **supported by** $p \in R^\ell \setminus \{0\}$ if $x'_i \succeq_i x_i \Rightarrow p \cdot x'_i \geq p \cdot x_i, \forall i \in I$

DEFINITION 1.7.5 : A feasible allocation $x \in X$ is **individually rational** if $x_i \succeq_i e_i, \forall i \in I$.

DEFINITION 1.7.6 : A feasible allocation $x \in X$ is **weakly Pareto optimal** if there is no feasible allocation $x' \in X$ such that $x'_i \succ_i x_i, \forall i \in I$.

DEFINITION 1.7.7 : An feasible allocation $x \in X$ is **Pareto optimal** if there is no feasible allocation $x' \in X$ such that $x'_i \succeq_i x_i, \forall i \in I$ and $x'_i \succ_i x_i$ for some $i \in I$.

THEOREM 1.7.1 : If x is supported by p and $\succeq$ is monotonic, then $p \geq 0$.

THEOREM 1.7.2 : Let (p, x) be a quasi-equilibrium. If $\succeq_i$ is reflexive for every i, then $p \cdot x_i = p \cdot e_i$ for every i. Moreover, if $\succeq_i$ is monotonic, then $p \geq 0$.

EXAMPLE 1.7.1 : When $\succeq_i$ is not strictly monotone and (p, x) is a quasi-equilibrium, it need not be the case that $p \gg 0$.
THEOREM 1.7.3 : If $\succeq_i$ is reflexive and continuous, and $e_i \in intX_i$ for every i, then $Q(\mathcal{E}) \subset W(\mathcal{E})$.