**Proof (for details of this proof see Debreu(1970):** W.l.o.g. assume that first consumer satisfies assumption A. Let $U = S \times L \times P^{m-1}$, an open set in $R^{\ell m}$. We define the function $F$ from $U$ to $R^{\ell m}$ by $F(e) = (\omega_1, \omega_2, \ldots, \omega_m)$ where $e = (p, w_1, \omega_2, \ldots, \omega_m)$ and

$$\omega_1 = f_1(p, w_1) + \sum_{i=2}^{m} f_i(p, p \cdot \omega_i) - \sum_{i=2}^{m} \omega_i$$

Notice that $\forall e \in U$, $p \cdot \omega_1 = w_1$. Also, given $\omega \in P^m$, the price vector $p$ belongs to $W(\omega)$ if and only if $F(p, p \cdot \omega_1, \omega_2, \ldots, \omega_m) = \omega$ and that the points of $W(\omega)$ are in one-to-one correspondence with the points of $F^{-1}(\omega)$. Since $F$ is continuously differentiable by Sard's theorem, the set $C$ of critical values of $F$ is null.
We now want to prove that $C \cap P^m$ is closed relative to $P^m$. To this end we establish that if $K$ is a compact subset of $P^m$, then $F^{-1}(K)$ is compact. This implies that if $E$ contained in $U$ is closed relative to $U$, then $F(E) \cap P^m$ is closed relative to $P^m$. Then, as a corollary we have $C \cap P^m$ is closed relative to $P^m$. If $\omega \in P^m$ is a regular value of $F$, then $F^{-1}(\omega)$ is finite. If $\omega \in P^m$ is such that $W(\omega)$ is infinite, then $\omega \in C$. Then, $C \cap P^m$ is null and so is its closure.

## 1.6 Stability of Walrasian Equilibrium

Uniqueness property is obtained under strong assumptions. With less restrictive assumptions we can have economies with multiple equilibria. This may be still satisfactory provided that all the equilibria are locally unique which is equal to the finiteness of equilibria when the set of equilibria is compact. Here, we want to show that almost every economy has a finite set of equilibria.

**DEFINITION 1.6.1 :** *Stability* means that aggregate excess demand goes down as prices go up.

**EXAMPLE 1.6.1 :** Suppose $e_1 = (1, 0), e_2 = (0, 1)$ and $p$ be the price of $x$, $q$ price of $y$ and $u_1(x, y) = min\{x, 2y\}, u_2(x, y) = min\{2x, y\}$. We can derive demand functions as follows. If $\alpha$ units of good $y$ is demanded then $2\alpha$ units of good $x$ are demanded. For agent 1, a budget line is $2\alpha p + \alpha q = p$. Thus $\alpha = p/(2p + q)$ and $D_1(p, q) = (2p/(2p + q), p/(2p + q))$. Similarly, $D_2(p, q) = (q/(p + 2q), 2q/(p + 2q))$. Then aggregate demand for $x$ is $E_x = 2p/(2p + q) + q/(p + 2q) - 1 = q(p - q)/(2q^2 + 5pq + 2p^2)$. Since denominator is always positive, as $p$ rises, $E_x$ increases so that there is an instability. Similarly, $E_y = p(q-p)/(2p^2+5pq+2q^2)$ so that there is an instability (See figure (a)).