Now, we will be more explicit about the expression "almost every". When we want to measure an interval on a line, the first measure we think of is to take the difference of the end points. What about the measure of more comlicated sets, such as union of intervals, or union of intervals and points etc...

Lebesgue Measure: For each set A of real numbers consider the countable collection $\{I_n\}$ of open intervals that cover A, that is, $A \subset \cup I_n$, and for each such collection consider the sum of the length of the intervals in the collection. Lebesgue measure of A, which is denoted by mA, is defined as the infimum of all such sums.

$$mA = \inf_{a \subset \cup I_n} \sum \ell(I_n)$$

where $\ell(I_n)$ represents the length of interval I_n.

EXAMPLE: There may exist sets with Lebesgue measure zero: e.g. set of rational numbers.

Sard's Theorem: If all the partial derivatives of F to the c^{th} order included, where $c > max(0, a - b)$, exist and are continuous, then the set of critical values of F has Lebesgue measure zero in R^b.

Let L be the set of strictly positive real numbers, P be the set of strictly positive vectors in R^ℓ where ℓ is the number of commodities, and S be the set of vectors in P for which the sum of the components is unity. There are m agents in the economy and agent i's demand function, f_i, is a function from $S \times L$ to $\overline{P}$ such that for every $(p, w_i) \in S \times L$, one has $p \cdot f_i(p, w_i) = w_i$.

Assumption A: If the sequence (p^q, w_i^q) in $S \times L$ converges to (p^0, w_i^0) in $(\overline{S} \setminus S) \times L$, then$|\ f_i(p^q, w_i^q)\ |$ converges to $+\infty$ (Every commodity is desired by agent i).

An economy is defined by $\omega \in P^m$. Given $\omega \in P^m$, an element p of S is an equilibrium price vector of the economy ω if

$$\sum_{i=1}^{m} f_i(p, p \cdot \omega_i) = \sum_{i=1}^{m} \omega_i.$$

We denote by $W(\omega)$ the set of p satisfying this equality. Finally we say that a set A is null if it has Lebesgue measure zero, also we say that a property holds almost everywhere if it holds outside of a null set.

Theorem: Given m continuously differentiable demand functions $(f_1, \ldots, f_m)$, if some f_i satisfies assumption A, then the set of $\omega \in P^m$ for which $W(\omega)$ is infinite has a null closure.