(b) $\forall i \in I, P_i(x^*) \cap \{x_i \in X_i : p^* \cdot x_i \leq p^* \cdot e_i\} = \emptyset$,

(c) $\sum_{i\in I} x_i^* \leq \sum_{i \in I} e_i$.

PROOF: Define $\bar{P}_{n+1} : \Delta \times X \to 2^\Delta$ by $\bar{P}_{n+1}(p, x) := \{q \in \Delta : q \cdot (\sum_{i\in I}(x_i - e_i)) > p \cdot (\sum_{i\in I}(x_i - e_i))\}$ and $\bar{A}_{n+1} : \Delta \times X \to 2^\Delta$ by $\bar{A}_{n+1}(p, x) := \Delta := X_{n+1}$. For each $i \in I$, define $\bar{P}_i : \Delta \times X \to 2^{X_i}$ by $\bar{P}_i(p, x) := P_i(x)$ and $\bar{A}_i : \Delta \times X \to 2^{X_i}$ by $\bar{A}_i(p, x) := \{x_i \in X_i : p \cdot x_i \leq p \cdot e_i\}$.

Then we have converted the exchange economy $\mathcal{E}$ to an abstract economy $\Gamma = \{(X_i , \bar{P}_i, \bar{A}_i) : i = 1, \ldots, n+1\}$ which satisfies all the conditions of the previous theorem.

Thus, there exists $(p^*, x^*) \in \Delta \times X$ such that

(i) $\forall i \in I, x_i^* \in \bar{A}_i(p^*, x^*)$, which is equivalent to (a),

(ii) $\forall i \in I, P_i(x^*) \cap \bar{A}_i(p^*, x^*) = \emptyset$, which is equivalent to (b),

(iii) $\bar{P}_{n+1}(p^*, x^*) \cap \bar{A}_{n+1}(p^*, x^*) = \emptyset$

However, (iii) implies that $\forall q \in \Delta, q \cdot z^* \leq p^* \cdot z^* \leq 0$, where $z^* = \sum_{i\in I}(x_i^* - e_i)$. Now suppose that $z^* \nleq 0$. Since $-z^* \notin R^\ell_+$, by separating hyperplane theorem, there exists $v \in R^\ell \setminus \{0\}$ such that $v \cdot (-z^*) < 0$. Without loss of generality, $v \in \Delta$. Thus $v \cdot z^* > 0$, which is a contradiction. Hence $z^* \leq 0$, i.e., $\sum_i x_i^* \leq \sum_i e_i$. □

1.5 Uniqueness of Walrasian Equilibrium

Uniqueness of the equilibrium is obtained under strong assumptions. With less restrictive assumptions we can have economies with multiple equilibria. This may be still satisfactory provided that all the equilibria are locally unique which is equivalent to the finiteness of equilibria when the set of equilibria is compact.
Here, we are going to show that "almost every" economy has a finite set of equilibria. That is, outside of a null closed subset of the space of economies, every economy has a finite set of equilibria. We will begin with some notation and some preliminary concepts.
Let $F : U \to R^b$ be a continuously differentiable function, where U is an open subset of R^a. A point x is a critical point of F if the Jacobian matrix of F at x has a rank smaller than b, and $y = F(x)$ is a critical value of F. If a point in R^b is not a critical value, then it is called a regular value.