By the Berge Maximum Theorem, for all i, φ_i is u.h.c and compact-valued. Also φ_i is nonempty since u_i is a continuous function defined on a compact set (Weierstrass Theorem). Moreover, for all i, φ_i, is convex-valued, since u_i is quasi-concave. Now define a new correspondence $\Phi : X \to 2^X$ by,

$$\Phi(x) = \Pi_{i=1}^n \varphi_i(\tilde{x}_i)$$

Notice now, that Φ carries all the properties of φ_i. Hence, Φ has a closed graph (since it is uhc and closed-valued and X is compact), is nonempty and convex-valued. Therefore, by the Kakutani fixed point theorem, there exists an $x^* \in X$ such that $x^* \in \Phi(x^*)$. It can be easily seen that the fixed point by construction is a Nash equilibrium. □

DEFINITION 1.4.5 : An **exchange economy** $\mathcal{E}$ is $\{(X_i, P_i, e_i) : i \in I\}$ where, for every $i \in I$,

(1) X_i is the consumption set of agent i,

(2) $P_i : X \to 2^{X_i}$ is the preference correspondence of agent i,

(3) $e_i \in X_i$ is the initial endowment of i.

DEFINITION 1.4.6 : An **equilibrium for the exchange economy** $\mathcal{E}$ is $(p^*, x^*) \in \Delta \times X$ such that

(a) $\forall i \in I, p^* \cdot x_i^* \leq p^* \cdot e_i$,

(b) $\forall i \in I, P_i(x^*) \cap \{x_i \in X_i : p^* \cdot x_i \leq p^* \cdot e_i\} = \emptyset$,

(c) $\sum_{i \in I} x_i^* = \sum_{i \in I} e_i$.

THEOREM 1.4.3 : Let $\mathcal{E} = \{(X_i, P_i, e_i) : i \in I\}$ be an exchange economy satisfying the following assumptions for each $i \in I$.

(1) X_i : is a nonempty compact convex subset of R^ℓ,

(2) P_i has an open graph in $X \times X_i$ and $x_i \notin coP_i(x), \forall x \in X$,

(3) $e_i \in intX_i$.

Then $\mathcal{E}$ has a free disposal equilibrium, i.e., there exist $(p^*, x^*) \in \Delta \times X$ such that

(a) $\forall i \in I, p^* \cdot x_i^* \leq p^* \cdot e_i$,