Then F_i is nonempty closed convex valued and upper-hemicontinuous. Define $\Psi : X \to 2^X$ by $\Psi(x) := \prod_i^n F_i(x)$. Then Ψ is nonempty closed convex valued and upper-hemicontinuous. Therefore, by the Kakutani fixed point theorem, there exists $x^* \in X$ such that $x^* \in \Psi(x^*)$. If for some i, $x^* \in V_i$, it follows from the definition of F_i that $x_i^* = f_i(x^*) \in \psi_i(x^*) \subset coP_i(x^*)$, which is a contradiction to (3). Thus for every i, $x^* \notin V_i$ which implies that $x_i^* \in A_i(x^*)$ and $\psi_i(x^*) = \emptyset$, i.e., $P_i(x^*) \cap A_i(x^*) = \emptyset$. Hence x^* is an equilibrium of Γ. □
Next we use the above Theorem to prove the existence of Nash equilibrium for a game in a normal form as a Corollary.

COROLLARY 1.4.1: Let $\mathcal{G} = \{(X_i, u_i) : i = 1, 2, ..., n\}$ be a **game in normal form** satisfying for all i the following assumptions:
i) X_i is compact, convex and non empty subset of R^l,
ii) $u_i : \Pi_{j=1}^n X_j \to R$ is quasi - concave and continuous.
Then $\mathcal{G}$ has a Nash equilibrium, i.e., there exists an $x^* \in X = \Pi_{i=1}^n X_i$ such that forall i,

$$u_i(x_1^*, ..., x_n^*) \geq u_i(x_1^*, ..., y_i, ..., x_n^*), \forall y_i \in X_i$$

PROOF: $\forall i$ set $A_i(x) = X_i$. Also, $\forall i$ define the correspondence $P_i : X \to 2^{X_i}$ by,

$$P_i(x_1, ..., x_n) = \{y_i \in X_i : u_i(x_1, ..., y_i, ..., x_n) > u_i(x_1, ..., x_n)\}$$

Hence, we have an abstract economy $\Gamma = \{(X_i, P_i, A_i) : i = 1, 2, ..., n\}$. We can easily verify the following: a) P_i has an open graph, b) P_i is convex - valued, c) $x_i \notin P_i(x_1, .., x_n), \forall x \in X$ and d) $A_i : X \to 2^{X_i}$ is non empty, closed - valued, convex - valued and continuous. Thus, Γ has an equilibrium, i.e., there exists an $x^* \in X$ such that,
i) $x_i^* \in A_i(x^*), \forall i$,
ii) $A_i(x^*) \cap P_i(x^*) = \emptyset, \forall i$.
From i) and ii) we can deduce that for all i, $x_i^* \in X_i$ and $P_i(x^*) = \emptyset$. That is,

$$\forall y_i \in X_i, u_i(x_1^*, ..., y_i, ..., x_n^*) \leq u_i(x_1^*, ..., x_n^*)$$

□
Now we provide an alternative proof of the existence of a Nash equilibrium in a normal form game (Corollary 1.4.1), by using the Berge Maximum Theorem and the Kakutani Fixed point Theorem.

PROOF: Let $X = \Pi_{i=1}^n X_i$ and $\tilde{X}_i = \Pi_{i \neq j} X_j$. For all i define $\varphi_i : \tilde{X}_i \to 2^{X_i}$ by,

$$\varphi_i(\tilde{x}_i) = \{y_i \in X_i : u_i(y_i, \tilde{x}_i) = \max_{z_i \in X_i} u_i(z_i, \tilde{x}_i)\}$$